Bellwork

Factor each of the following expressions completely:

1. $-x^2 + 3x + 70$

$-1 \quad -1 \quad -1$

Factor out 1(70) = 70: 1 and 70, 2 and 35, 5 and 14, 7 and 10

$$x^2 - 3x - 70$$
$$x^2 + 7x - 10x - 70$$
$$(x^2 + 7x) - (10x + 70)$$
$$x(x + 7) - 10(x + 7)$$
$$-1(x - 10)(x + 7)$$

2. $15x^2 + 13x + 2$

Factor out 15(2) = 30: 1 and 30, 2 and 15, 3 and 10, 5 and 6

$$15x^2 + 3x + 10x + 2$$
$$(15x^2 + 3x) + (10x + 2)$$
$$3x(5x + 1) + 2(5x + 1)$$
$$(3x + 2)(5x + 1)$$

Review on Factoring Expressions

Equation is given in standard form:

$$y = ax^2 + bx + c$$

Step 1: Multiply a times c.
Step 2: Factor out ac by completing the U.
Step 3: Select the factors that combine by the second sign to get the middle number.
Step 4: Remove the middle term and replace it with the 2 terms found in Step 3.
Step 5: Group first 2 terms, and last 2 terms.
Step 6: Factor out the GCF's
Step 7: Write the factored version and check!

Basic Concepts VS. Specific Concepts

When graphing, if possible, we needed to see:

1. x-intercept(s)
2. y-intercept
3. Vertex

Now algebraically, we need to identify:

1. The zeros of the function
2. The Axis of Symmetry
3. The Extreme Value

Definitions

Zeros:

The zeros of a function are the values of x that form the x-intercepts.

Example: (-5,0) means the zero is -5.

Axis of Symmetry:

Still finding this using $x = -b/2a$

Extreme Value:

Defining if the vertex is a Maxima or Minima, then identifying its value .

Identifying the Zeros of a Function

IF the expression is UNFACTORABLE:

Plug into the following expression and simplify where possible...

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

IF the expression can be factored:

Step 1: Factor the expression completely.

Step 2: Set each factor = 0.

Step 3: Solve for the variable.

NOTE: These could also be referred to as the **ROOTS** of the function.

Examples

Identify the zeros of the given equations:

1. $f(x) = 2(x+5)(x-6)$

Zeros means f(x) becomes 0.

$0 = 2(x+5)(x-6)$

$x + 5 = 0$, $\quad -5 \quad -5$, $\boxed{x = -5}$

$x - 6 = 0$, $\quad +6 \quad +6$, $\boxed{x = 6}$

2. $h(t) = -3t^2 + 5t + 1$

This is not factorable. Plug into

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

a = -3, b = 5, & c = 1

$$\frac{-5 \pm \sqrt{(5)^2 - 4(-3)(1)}}{2(-3)}$$

$$\boxed{= \frac{-5 \pm \sqrt{37}}{-6}}$$

Examples

Find the Axis of Symmetry for each of the given functions:

3. $A(t) = 2t^2 + 8t - 9$

$a = 2$, $b = 8$, & $c = -9$

AOS:

$$t = \frac{-b}{2a} = \frac{-(8)}{2(2)} = \frac{-8}{4} = -2$$

$$\boxed{t = -2}$$

4. $s(t) = 5t^2 - 7t + 51$

$a = 5$, $b = -7$, & $c = 51$

AOS:

$$t = \frac{-b}{2a} = \frac{-(-7)}{2(5)} = \frac{7}{10}$$

$$\boxed{t = \frac{7}{10}}$$

Identifying the Extreme Value of a Function

Procedure:

Step 1: Identify if the function has a Maxima or a Minima.

Leading Coefficient is...

+ then we have a Minima

- then we have a Maxima

Step 2: Find the AOS of the function.

Step 3: Plug the AOS into the function.

Examples

Identify the Extreme Value in each of the following functions:

5. $y = x^2 + 6x - 8$

Since Leading Coefficient is POSITIVE: ***Minima***

AOS:

$$x = \frac{-(6)}{2(1)} = \frac{-6}{2} = -3$$

Plugging in:

$x^2 + 6x - 8$

$(-3)^2 + 6(-3) - 8$

$9 - 18 - 8$

$-9 - 8$

-17

6. $y = -3x^2 + 9x + 7$

Since Leading Coefficient is NEGATIVE: ***Maxima***

AOS:

$$x = \frac{-(9)}{2(-3)} = \frac{-9}{-6} = \frac{3}{2}$$

Plugging in:

$-3x^2 + 9x + 7$

$-3(3/2)^2 + 9(3/2) + 7$

$-3(9/4) + 27/2 + 7$

$-27/4 + 54/4 + 28/4$

55/4

Example

Without graphing the following functions identify each of the following:

A. The zeros of the function.

B. The axis of Symmetry.

C. The extreme value.

7. $A(x) = -x^2 - 6x + 2$

Finding the Zeros:

$-1(x^2 + 6x - 2)$

Notice ac = 1(-2) = -2

There are no factors of 2 that will subtract to get +6.

Therefore,

The Function is Unfactorable.

Plug into that expression.

$$\frac{-(-6) \pm \sqrt{(-6)^2 - 4(-1)(2)}}{2(-1)} = \boxed{\frac{6 \pm \sqrt{44}}{-2}}$$

Finding the AOS:

$$x = \frac{-b}{2a} = \frac{-(-6)}{2(-1)} = \frac{6}{-2} = -3$$

$\boxed{\text{AOS is } x = -3}$

Extreme Value:

Maxima/Minima?

Since the Leading Coefficient is -1, the negative makes the graph go downward. Thus we have a *Maxima.*

Maxima Value

$-x^2 - 6x + 2$

$-(-3)^2 - 6(-3) + 2$

$-(9) + 18 + 2$

$9 + 2$

$\boxed{7}$

Example

Without graphing the following functions identify each of the following:

A. The zeros of the function.

B. The axis of Symmetry.

C. The extreme value.

8. $h(t) = -2t^2 + 3t - 1$

Finding the Zeros:

$h(t) = -1(2t^2 - 3t + 1)$

So 2(1) = 2 factors that add to get -3 are -2 + (-1).

$h(t) = -1(2t^2 - 2t - 1t + 1)$

$h(t) = -1[2t(t - 1) - 1(t - 1)]$

$h(t) = -1(2t - 1)(t - 1)$

Therefore…

$2t - 1 = 0$ $\quad$ $+1 \quad +1$

$\frac{2t}{2} = \frac{1}{2}$

$\boxed{t = 1/2}$

$t - 1 = 0$ $\quad$ $+1 \quad +1$

$\boxed{t = 1}$

Finding the AOS:

$$t = \frac{-(3)}{2(-2)} = \frac{-3}{-4} = \frac{3}{4}$$

$$\boxed{\text{AOS is } t = \frac{3}{4}}$$

Extreme Value:

Maxima/Minima?

Since the Leading Coefficient is -2, the negative makes the graph go downward. Thus we have a *Maxima.*

Maxima Value

$-2(3/4)^2 + 3(3/4) - 1$

$-2(9/16) + 9/4 - 1$

$-18/16 + 9/4 - 1$

$\boxed{1/8}$