Math II: 2nd Semester Final Midpoint Test

Unit 3: Comparing Functions – Modeling and Transformations

Find the equation of the line that goes through the given points.

1. A linear equation through the points (-48, -9) and (8, -2).
   (4 points)

$x_1 \; y_1 \quad x_2 \; y_2$

Method 1

① $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-2-(-9)}{8-(-48)} = \frac{7}{56} = \frac{1}{8}$

② $y = mx + b$

$-2 = \frac{1}{8}\left(\frac{8}{1}\right) + b$

$-2 = 1 + b$

$-1 \quad -1$

$-3 = b$

$\boxed{③\ y = \frac{1}{8}x - 3}$

Method 2

$y = mx + b$

$-9 = m(-48) + b$

$-9 = -48m + b$

$+48m \quad +48m$

$48m - 9 = b$

$48\left(\frac{1}{8}\right) - 9 = b$

$b = 6 - 9 = -3$

$y = mx + b$

$-2 = m(8) + b$

$-2 = 8m + b$

$-2 = 8m + 48m - 9$

$-2 = 56m - 9$

$+9 \quad +9$

$\frac{7}{56} = \frac{56m}{56}$

$\boxed{\frac{1}{8} = m}$

$\boxed{y = \frac{1}{8}x - 3}$

2. A quadratic equation through the points (-2, 57), (2, 17), and (7, 372).
   (6 points)

$y = ax^2 + bx + c$

$57 = (-2)^2a + (-2)b + c$

$57 = 4a - 2b + c$

$57 = 4a - 2b - 4a - 2b + 17$

$57 = -4b + 17$

$-17 \quad -17$

$\frac{40}{-4} = \frac{-4b}{-4}$

$\boxed{-10 = b}$

Plug in b to solve for a

$17 = (2)^2a + (2)b + c$

$17 = 4a + 2b + c$

$-4a - 2b \quad -4a - 2b$

$\boxed{-4a - 2b + 17 = c}$

$-4(9) - 2(-10) + 17 = c$

$-36 + 20 + 17 = c$

$-16 + 17 = c$

$\boxed{1 = c}$

Answer:

$y = 9x^2 - 10x + 1$

$372 = (7)^2a + (7)b + c$

$372 = 49a + 7b + c$

$372 = 49a + 7b - 4a - 2b + 17$

$372 = 45a + 5b + 17$

$-17 \quad -17$

$\frac{355}{5} = \frac{45a}{5} + \frac{5b}{5}$

$\boxed{71 = 9a + b}$

$71 = 9a + (-10)$

$71 = 9a - 10$

$+10 \quad +10$

$\frac{81}{9} = \frac{9a}{9}$

$\boxed{9 = a}$

Plug in a & b

3. An exponential equation through the points (-5, 192) and (-1, 12).
   (4 points)

$y = ab^x \quad x \; y \quad x \; y$

$192 = ab^{-5}$

$12 = ab^{-1}$

Smaller Exp.
Denominator

$\frac{12 = \cancel{a}b^{-1-(-5)}}{192 = \cancel{a}b^{-5}}$

becomes $\sqrt[4]{\frac{1}{16}} = \sqrt[4]{b^4}$ Gives $\boxed{\frac{1}{2} = b}$

$12 = ab^{-1}$

$12 = a\left(\frac{1}{2}\right)^{-1}$

$\frac{12}{2} = \frac{a(2)}{2}$

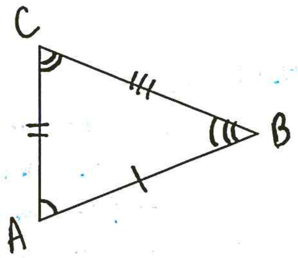
$\boxed{6 = a}$

Answer

$y = 6\left(\frac{1}{2}\right)^x$

Unit 4: Similarities

4. Given the following triangles, provide **CONGRUENCE** statements for the angles and sides that will show that Δ ABC ≅ Δ XYZ (6 points):

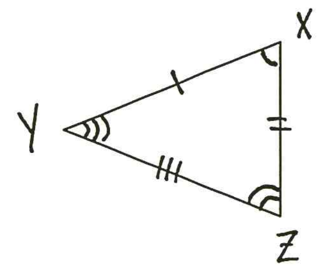


3 sides

$\overline{AB} \cong \overline{XY}$
$\overline{AC} \cong \overline{XZ}$
$\overline{BC} \cong \overline{YZ}$

3 Angles

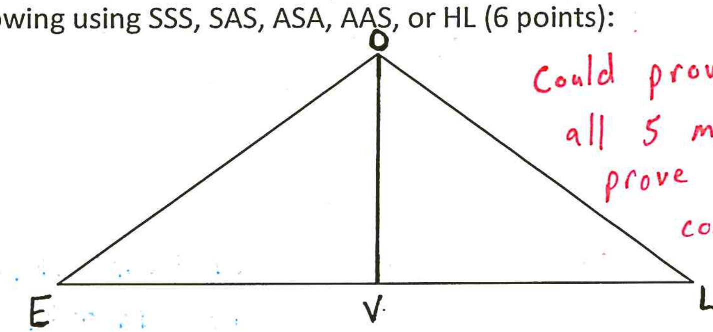
$\angle A \cong \angle X$
$\angle C \cong \angle Z$
$\angle B \cong \angle Y$



5. Write a two column proof for the following using SSS, SAS, ASA, AAS, or HL (6 points):

Given: OV ⊥ LE
OV bisects ∠O
Δ LOE is an isosceles

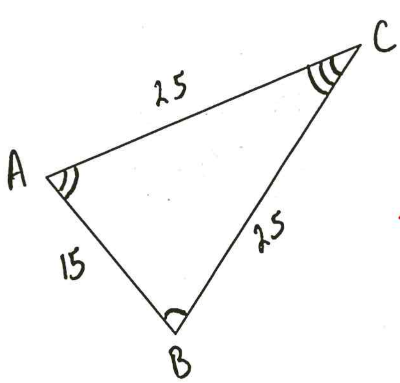
Prove: Δ LOV ≅ Δ EOV



Could prove using all 5 methods to prove these are congruent.

Possible Statements	Reasons
Since $\overline{OV} \perp \overline{LE}$	
① ∠OVE & ∠OVL are Right Angles	① Definition of ⊥
(2A) ∠OVE ≅ ∠OVL	(2A) Right Angles congruence theorem
(2B) Δ OVE & Δ OVL are Right Δ's	(2B) Definition Right Δ
Since $\overline{OV}$ bisects ∠O	
③ ∠VOE ≅ ∠VOL	③ Definition of bisects
Since ΔLOE is an isosceles Δ	
④ $\overline{OE} \cong \overline{OL}$	④ Definition of Isosceles Δ
⑤ ∠E ≅ ∠L	⑤ Definition of Isosceles Δ
Could say ⑥ $\overline{OV} \cong \overline{OV}$	⑥ Reflexive Property

6. Given the following triangles, provide **SIMILARITY** statements for the angles and sides that will show that Δ ABC ~ Δ XYZ (7 points):



3 Angles

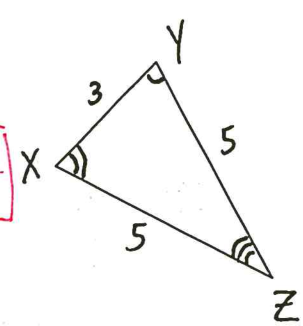
$\angle B \cong \angle Y$
$\angle A \cong \angle X$
$\angle C \cong \angle Z$

Side RATIOS

$$\boxed{\frac{AB}{XY} = \frac{BC}{YZ} = \frac{AC}{XZ} = \frac{5}{1}}$$

$\frac{15}{3}$ $\frac{25}{5}$ $\frac{25}{5}$

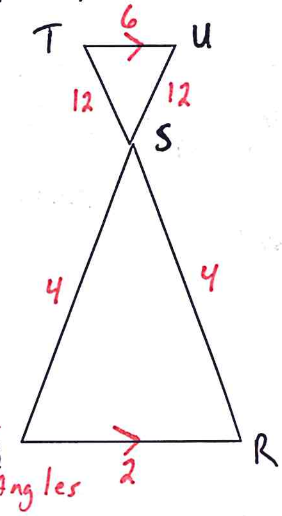
$\frac{5}{1}$ $\frac{5}{1}$ $\frac{5}{1}$



7. Write a two column proof for the following using AA, SSS, or SAS (6 points):

Given: QR || TU
QR = 2
QS = 4
RS = 4
ST = 12
SU = 12
TU = 6

May prove using all 3 methods.



Prove: Δ SQR ~ Δ SUT

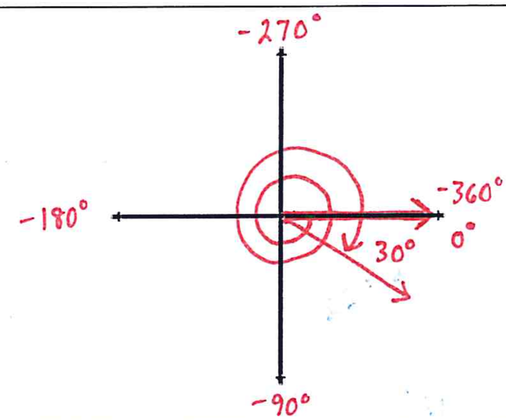
Possible Statements	Reasons.
① $\angle QSR \cong \angle UST$	① Vertical Angles
Since $\overline{QR} \parallel \overline{TU}$	
② $\angle Q \cong \angle U$	② Alternate Interior Angles.
③ $\angle R \cong \angle T$	③ Alternate Interior Angles.
Sides Ratios	
④ $\frac{SQ}{SU} = \frac{4}{12} = \frac{1}{3}$	④ Divide Top & Bottom by 4
⑤ $\frac{QR}{UT} = \frac{2}{6} = \frac{1}{3}$	⑤ Divide Top & Bottom by 2
⑥ $\frac{SR}{ST} = \frac{4}{12} = \frac{1}{3}$	⑥ Divide Top & Bottom by 4
⑦ $\frac{SQ}{SU} = \frac{QR}{UT} = \frac{SR}{ST} = \frac{1}{3}$	⑦ Transitive Property

Unit 5: Right Triangles and Trigonometry

8. Given an angle of -750°:

A. Draw the angle in standard form (1 point).

$$\begin{array}{rl} -750° & \\ +\ 360 & \text{1 rotation} \\ \hline -390° & \\ +360° & \text{2 rotations} \\ \hline -30° & \text{extra} \end{array}$$



B. Identify one positive coterminal angle (1 point):

$-30° + 360° = \boxed{330°}$

C. Identify one negative coterminal angle (1 point):

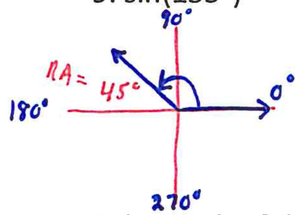
$\boxed{-390°}$ OR $\boxed{-30°}$

D. Find the reference angle of the given angle (1 point).

$\boxed{30°}$

Evaluate each of the following using the Hand Trick (2 points each):

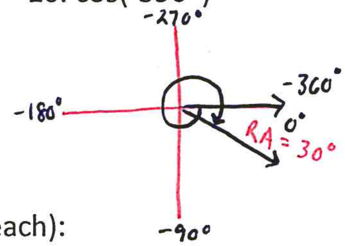
9. $\sin(135^{o})$



$$\boxed{\frac{\sqrt{2}}{2}}$$

Terminal side in Q II
sin is +

10. $\cos(-390^{O})$

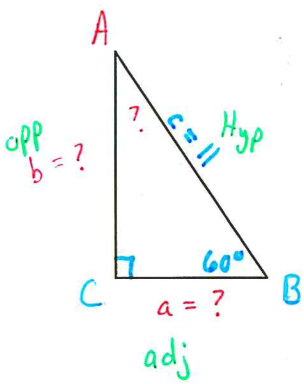


$$\boxed{\frac{\sqrt{3}}{2}}$$

Terminal side in Q IV
cos is +

Solve each of the following triangles (3 points each):

11. Given the RIGHT triangle with right $\angle$ C
$\angle B = 60^{o}$ & $c = 11$



Finding $\angle A$

$\angle A = 180^{\circ} - 90^{\circ} - 60^{\circ}$

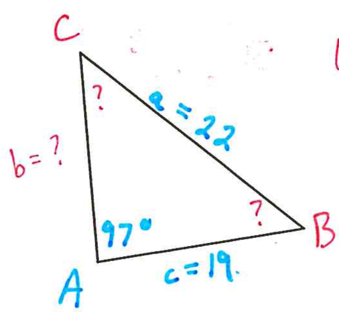
$\boxed{\angle A = 30^{\circ}}$

Finding the remaining sides (Using $\angle B$)

SOH - CAH - TOA

$11 \cdot \sin 60 = \frac{b}{11} \cdot 11$	$11 \cdot \cos 60 = \frac{a}{11} \cdot 11$	$\cancel{\tan 60 = \frac{b}{a}}$
$11 \sin 60 = b$	$11 \cos 60 = a$	Can't use since missing 2 variables
$11\left(\frac{\sqrt{3}}{2}\right) = b$	$11\left(\frac{\sqrt{1}}{2}\right) = a$	
$\frac{11\sqrt{3}}{2} = b$	$11\left(\frac{1}{2}\right) = a$	
$\boxed{9.5 \approx b}$	$\boxed{5.5 = a}$	

12. Given the Non-Right Triangle:
$\angle A = 97^{o}$, $a = 22$, & $c = 19$



Law of Cosines requires a corner of given information.
Use Law of Sines:

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

$$\frac{\sin 97}{22} = \frac{\sin B}{b} = \frac{\sin C}{19}$$

since the middle has 2 variables use outside first to find $\angle C$.

① $19 \cdot \frac{\sin 97}{22} = \frac{\sin C}{19} \cdot 19$

$\sin^{-1}\left(\frac{19 \sin 97}{22}\right) = \sin^{-1}(\sin C)$

$\boxed{59^{o} \approx \angle C}$

② Use $\angle C$ to find $\angle B$

$\angle B = 180^{\circ} - \angle A - \angle C$

$\angle B \approx 180^{\circ} - 97^{\circ} - 59^{\circ}$

$\boxed{\angle B \approx 24^{o}}$

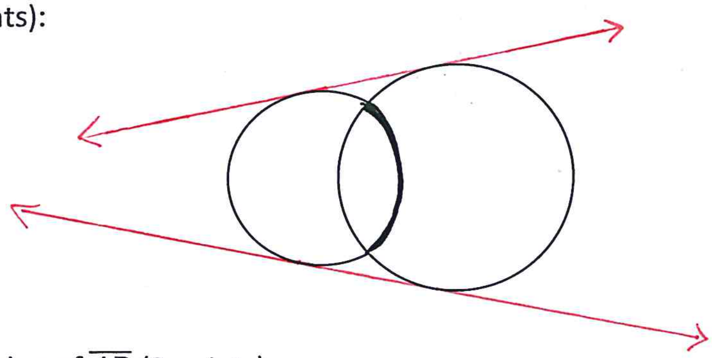
③ $\frac{\sin 97}{22} = \frac{\sin 24}{b}$

$\frac{b \cdot \sin 97}{\sin 97} = \frac{22 \sin 24}{\sin 97}$

$\boxed{b \approx 9.0}$

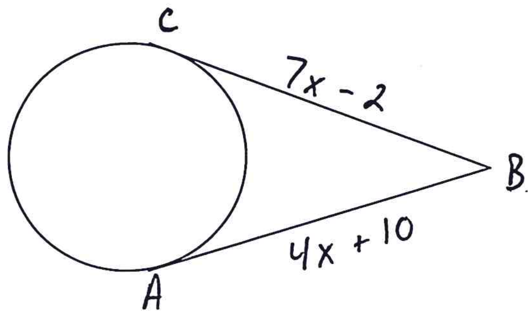
Unit 7: Circles:

13. State the number of common tangents between the given circles, and then draw them (2 points):



2 common tangents

14. Find the value of $\overline{AB}$ (2 points).



Solve for x

$7x - 2 = 4x + 10$
$-4x \;\; +2 \quad -4x \;\; +2$

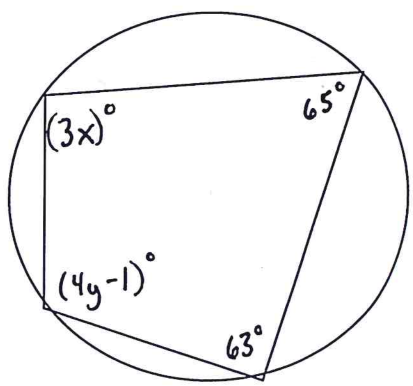
$\frac{3x}{3} = \frac{12}{3}$

$x = 4$

Plug in to find $\overline{AB}$

$4x + 10$
$4(4) + 10$
$16 + 10$
$\boxed{26}$

15. Find the value of all variables (2 points):



$3x + 63 = 180$
$\quad -63 \quad -63$

$\frac{3x}{3} = \frac{117}{3}$

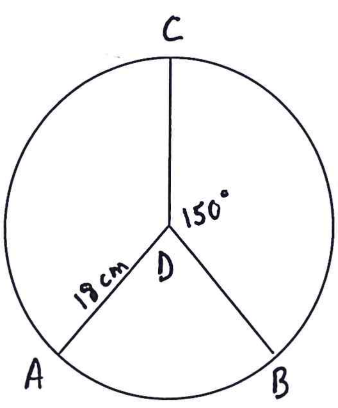
$\boxed{x = 39}$

$4y - 1 + 65 = 180$
$4y + 64 = 180$
$\quad -64 \quad -64$

$\frac{4y}{4} = \frac{116}{4}$

$\boxed{y = 29}$

16. In the following ∠ ADC ≅ ∠BDC. Find the length of $\widehat{AB}$ (1 point).



Length $\widehat{AB} = \frac{m\widehat{AB}}{360} \cdot 2\pi r$

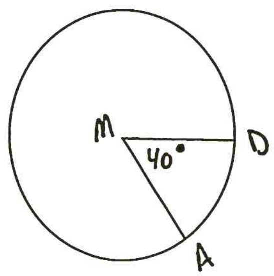
Length $\widehat{AB} = \frac{60}{360} \cdot 2\pi(18)$

$= \frac{1}{6} \cdot 36\pi$

$= \boxed{6\pi}$

$\angle ADB = 360 - 150 - 150$
$\angle ADB = 60 = m\widehat{AB}$

17. Find the area of circle M, given the area of the sector is 105 in² (1 point)



$$A_{sec} = \frac{m\widehat{AB}}{360} \cdot \pi r^2$$

$$\frac{360}{40} \cdot 105 = \frac{40}{360} \cdot A_{circle} \cdot \frac{360}{40}$$

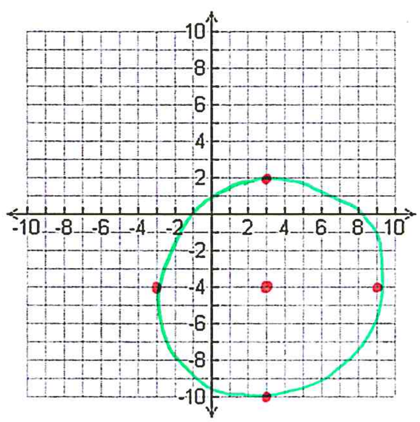
$$\boxed{945 \text{ in}^2 = A_{circle}}$$

18. Given the equation: $(x - 3)^2 + (y + 4)^2 = 36$

A. Identify the center of the circle (1 point): $(3, -4)$

B. Identify the radius of the circle (1 point): $\sqrt{36} = \boxed{6}$

C. Graph the circle (1 point):



19. Given the equation:
$x^2 + y^2 + 2x + 6y - 15 = 0$

A. Rewrite the equation into Standard Form (1 point):

$$x^2 + 2x + \left(\frac{2}{2}\right)^2 + y^2 + 6y + \left(\frac{6}{2}\right)^2 = 15 + \left(\frac{2}{2}\right)^2 + \left(\frac{6}{2}\right)^2$$

$$x^2 + 2x + (1)^2 + y^2 + 6y + (3)^2 = 15 + (1)^2 + (3)^2$$

$$(x+1)^2 + (y+3)^2 = 15 + 1 + 9$$

$$\boxed{(x+1)^2 + (y+3)^2 = 25}$$

A. Identify the center of the circle (1 point):
$(-1, -3)$

B. Identify the radius of the circle (1 point):
$\sqrt{25} = \boxed{5}$

D. Graph the circle (1 point)