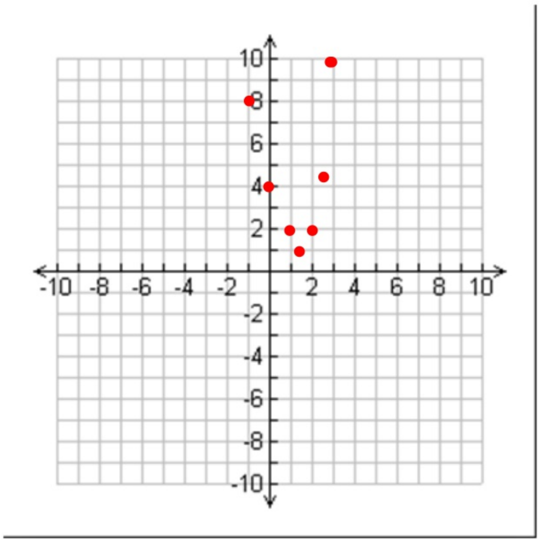
Bellwork

Graph the given points and classify the model as linear, exponential, or quadratic.

(-1,8), (0, 4), (1, 2)
(1.5, 1), (2, 2), (2.5, 4.5),
(3, 10)



Creating Quadratic Equations

Overview:

- Need to have 3 points
- Set up a system of 3 equations/ 3 unknowns
- Solve the system of equations

Methods Used:

- Substitution
- Elimination
- Combination of the two

Setting up the System of Equations

Recall the general Quadratic Equation:

$$y = ax^2 + bx + c$$

Recall what an Ordered Pair represents:

$$(x, y)$$

Set up 3 Equations using one point per equation:

(x_1 , y_1) gives $y_1 = a(x_1)^2 + b(x_1) + c$

(x_2 , y_2) gives $y_2 = a(x_2)^2 + b(x_2) + c$

(x_3 , y_3) gives $y_3 = a(x_3)^2 + b(x_3) + c$

Example

Set up the system of equations to find the Quadratic Model for the given set of points:

1. (-2, -17), (1, 7), & (5, -17)

Set up:

$$-17 = a(-2)^2 + b(-2) + c$$
$$7 = a(1)^2 + b(1) + c$$
$$-17 = a(5)^2 + b(5) + c$$

Simplified:

$$-17 = 4a - 2b + c$$
$$7 = a + b + c$$
$$-17 = 25a + 5b + c$$

Solving by Substitution

Step 1: Solve ONE of the equations for one of the variables a, b, or c.

(Suggestion: Solve for c)

Let's use... $-17 = 25a + 5b + c$

So we need to do:

$$\begin{array}{rl} & -17 = 25a + 5b + c \\ \underline{-25a - 5b} & \underline{\quad -25a - 5b \quad} \\ & -25a - 5b - 17 = 0 + 0 + c \\ & -25a - 5b - 17 = c \end{array}$$

Step 2: Plug the expression that c is equal to into the other two equations for c. ***Simplify***

We are going to use:

$$c = -25a - 5b - 17$$

And we will SUBSTITUTE that into the other two equations for c.

$-17 = 4a - 2b + c$	$7 = a + b + c$
$-17 = 4a - 2b + (-25a - 5b - 17)$	$7 = a + b + (-25a - 5b - 17)$
$-17 = 4a - 2b - 25a - 5b - 17$	$7 = a + b - 25a - 5b - 17$
$-17 = 4a - 25a - 2b - 5b - 17$	$7 = a - 25a + b - 5b - 17$
$-17 = -21a - 7b - 17$	$7 = -24a - 4b - 17$
$+17 \qquad\qquad +17$	$+17 \qquad\qquad +17$
$0 = -21a - 7b$	$24 = -24a - 4b$

Step 3: Check and see if the simplified equations can be simplified any more by dividing all the numbers by a common number.

$$\frac{0}{-7} = \frac{-21a}{-7} - \frac{7b}{-7}$$

Everything can be divided by -7 here

Giving us: **0 = 3a + b**

$$\frac{24}{-4} = \frac{-24a}{-4} - \frac{4b}{-4}$$

Everything can be divided by -4 here

Giving us: **-6 = 6a + b**

Step 4: Solve one of the two equations that are remaining for one of the variables.

Let's use:

$0 = 3a + b$

Why?

Since we have just 1 b it can be solved for b with avoiding any fractions making calculations easier.

$$
\begin{array}{rcl}
0 & = & 3a + b \\
\underline{-3a} & & \underline{-3a\quad\quad} \\
-3a & = & b
\end{array}
$$

Step 5: Take the expression you have found for b and plug it into the other equation that you did not solve for b.
Solve for the remaining variable.

We are going to use:

$b = -3a$

And we will SUBSTITUTE that into the other equation for b.

$-6 = 6a + b$
$-6 = 6a + (-3a)$
$-6 = 6a - 3a$
$-6 = 3a$ ⟵ Divide both sides by 3
$-2 = a$

Step 6: Going backwards now...

Plug a into one of the equations from Step 4, and solve for the other variable in one of those equations.

Since $a = -2$

Equation from Step 4: $b = -3a$

$$b = -3(-2)$$

$$b = 6$$

Step 7: Plug the two solved for variables into ONE of the equations from Step 1.

Now: $a = -2$
$b = 6$

Plugging into the equation from Step 1:

$$c = -25a - 5b - 17$$
$$c = -25(-2) - 5(6) - 17$$
$$c = 50 - 30 - 17$$
$$c = 20 - 17$$
$$c = 3$$

Step 8: Write the equation plugging in the values for a, b, and c into the general Quadratic Equation.

Now: $a = -2$
$b = 6$
$c = 3$

Therefore the equation would become:

$$y = ax^2 + bx + c$$

$$y = (-2)x^2 + (6)x + (3)$$

$$\boxed{y = -2x^2 + 6x + 3}$$ ← ANSWER!

Examples to try:

1. (-4, 54), (2, 12), & (5, 72)

2. (-3, -36), (1, -8), & (8, -421)

The Answers to the Examples

1. $y = 3x^2 - x + 2$

2. $y = -6x^2 - 5x + 3$

Solving by Elimination

Step 1: Write the equations so that the terms with a, the terms with b, the terms with c, the equal signs, and the constants are aligned.

2. (-5, 141), (-3, 55), & (1, 3)

Set-up:

$$141 = a(-5)^2 + b(-5) + c$$
$$55 = a(-3)^2 + b(-3) + c$$
$$3 = a(1)^2 + b(1) + c$$

Simplified:

$$141 = 25a - 5b + c$$
$$55 = 9a - 3b + c$$
$$3 = a + b + c$$

Step 2: Choose ONE equation to eliminate one of the variables. (You choose which variable that you are going to try and eliminate.)

Let's choose:

$3 = a + b + c$

Why? The choice is yours. I typically just go with the last equation.

Step 3: Subtract the equation you chose in Step 2 from each of the other two equations.

$$\begin{array}{r} 141 = 25a - 5b + c \\ \underline{-(3 = a + b + c)} \\ 138 = 24a - 6b \end{array}$$

$$\begin{array}{r} 55 = 9a - 3b + c \\ \underline{-(3 = a + b + c)} \\ 52 = 8a - 4b \end{array}$$

Step 4: We need to get one of the remaining variables to have the same number in front of them, but with opposite signs. (For example: -5b and 5b)

Again the choice is yours!
For the purpose of these notes we will choose the b terms.

Suggestion: Multiply each equation by the coefficient of the variable in the other equation!
The only difference is one may need to be negative if the signs are the same.

$4(138 = 24a - 6b) \longrightarrow 552 = 96a - 24b$

$-6(52 = 8a - 4b) \longrightarrow -312 = -48a + 24b$

Notice now the b terms are opposites!

Step 5: Add the two altered equations together and solve for the remaining variable.

$$
\begin{aligned}
552 &= 96a - 24b \\
\underline{-312} &\underline{= -48a + 24b} \\
\frac{240}{48} &= \frac{48a}{48}
\end{aligned}
$$

$$5 = a$$

Step 6: Plug the value of the variable solved for in Step 5 into one of the equations from Step 3 or Step 4.
Solve for the remaining variable.

Let's choose this one:

$52 = 8a - 4b$

$52 = 8(5) - 4b$

$52 = 40 - 4b$

$\underline{-40 \quad -40 \qquad}$

$\frac{12}{-4} = \frac{-4b}{-4}$

$-3 = b$

Step 7: Take the value for the variable solved for in Step 5 and in Step 6, and plug them into one of the equations from Step 1.
Solve for the remaining variable.

Equation from Step 1 that we used:

$3 = a + b + c$

Variables we found:

$a = 5$

$b = -3$

Plug in and solve...

$3 = 5 + (-3) + c$

$3 = 5 - 3 + c$

$3 = 2 + c$

$\underline{-2 \quad -2 \quad\quad}$

$1 = c$

Step 8: Write the equation plugging in the values for a, b, and c into the general Quadratic Equation.

We found the variables to be:

$a = 5$

$b = -3$

$c = 1$

Plugging into the equation:

$$y = ax^2 + bx + c$$

$$\boxed{y = 5x^2 - 3x + 1}$$ ← **Answer!**

Examples to try:

1. (-6, 6), (2, -2), & (8, 34)

2. (-8, -227), (-3, -22), & (5, -110)

The Answers to the Examples

1. $y = 0.5x^2 + x - 6$

2. $y = -4x^2 - 3x + 5$