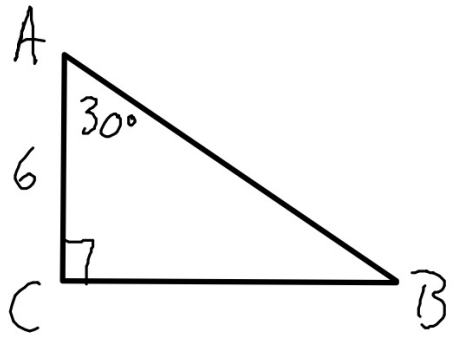
Bellwork

Solve the following right triangle given the information:

1. $\angle A = 30^\circ$ & b = 6



Review on Solving Triangles

Given:

1. Right Triangles - Given 2 pieces of info.
    A. 2 sides
    B. 1 side and 1 angle
2. Any other triangle - Given 3 pieces of info
    A. 2 sides and 1 angle
    B. 1 side and 2 angles

Need:

Find all remaining sides and angles that are not given.

The Law of Sines

$$\frac{\text{Sin } A}{a} = \frac{\text{Sin } B}{b} = \frac{\text{Sin } C}{c}$$

OR

$$\frac{a}{\text{Sin } A} = \frac{b}{\text{Sin } B} = \frac{c}{\text{Sin } C}$$

Word of Caution

When given 2 sides and a non-included angle there are several possible answers:

A. No solutions

B. One solution

C. Two solutions

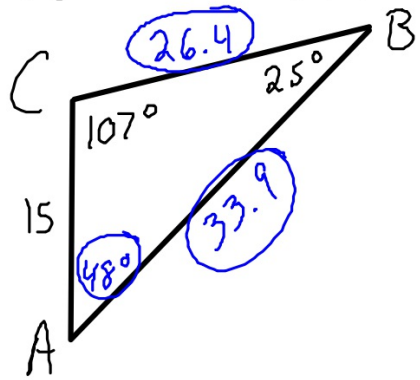
Focus:

We will focus on the situations where there is at least one solution. I will find both of them and if you have a different answer than the first I give in class ask about it.

Example

Solve △ ABC:

1. ∠B = 25°, ∠C = 107°, & b = 15



Given 2 angles, find A:

∠A = 180° - 107° - 25°

∠A = 48°

$15 \sin 48 = a \sin 25$

$$a = \frac{15 \sin 48}{\sin 25} \approx 26.4$$

$$\frac{\sin 48}{a} = \frac{\sin 25}{15} = \frac{\sin 107}{c}$$

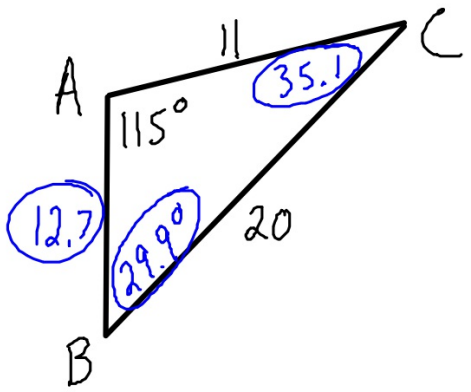
$c \sin 25 = 15 \sin 107$

$$c = \frac{15 \sin 107}{\sin 25} \approx 33.9$$

Example

Solve $\triangle ABC$:

2. $\angle A = 115°$, $a = 20$, & $b = 11$



$$\frac{\sin 115}{20} = \frac{\sin B}{11} = \frac{\sin C}{c}$$

Find $\angle B$: $11 \sin 115 = 20 \sin B$

$$\frac{11 \sin 115}{20} = \sin B$$

$$\angle B = \sin^{-1}\left(\frac{11 \sin 115}{20}\right)$$

$\angle B \approx 29.9°$

Find $\angle C$:

$\angle C = 180 - 115 - 29.9$

$\angle C = 35.1°$

Now: $\frac{\sin 115}{20} = \frac{\sin 35.1}{c}$

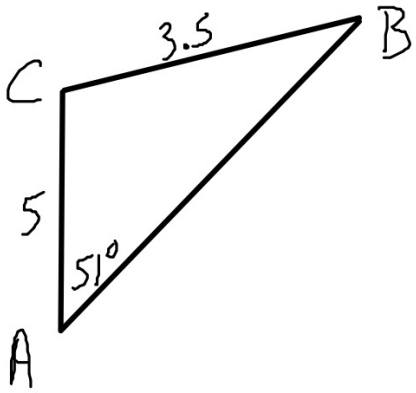
$c \sin 115 = 20 \sin 35.1$

$$c = \frac{20 \sin 35.1}{\sin 115} \approx 12.7$$

Example

Solve △ ABC:

3. ∠ A = 51°, a = 3.5, & b = 5



$$\frac{\sin 51}{3.5} = \frac{\sin B}{5} = \frac{\sin C}{c}$$

Find ∠B: $5 \sin 51 = 3.5 \sin B$

$$\sin B = \frac{5 \sin 51}{3.5}$$

$$\angle B = \sin^{-1}\left(\frac{5 \sin 51}{3.5}\right)$$

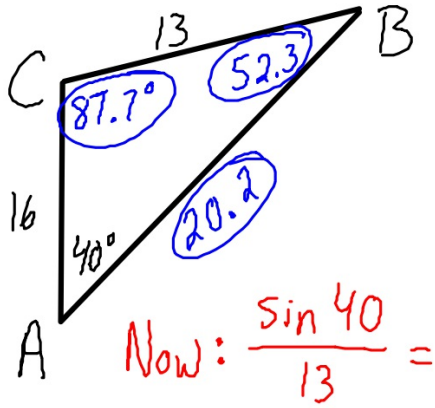
= Error

No Triangle Possible

Example

Solve △ ABC:

4. ∠A = 40°, a = 13, b = 16



$$\frac{\sin 40}{13} = \frac{\sin B}{16} = \frac{\sin C}{c}$$

Find B: $16 \sin 40 = 13 \sin B$

$$\sin B = \frac{16 \sin 40}{13}$$

$$\angle B = \sin^{-1}\left(\frac{16 \sin 40}{13}\right)$$

$\angle B \approx 52.3°$

Now: $\frac{\sin 40}{13} = \frac{\sin 87.7}{c}$

$c \sin 40 = 13 \sin 87.7$

$$c = \frac{13 \sin 87.7}{\sin 40}$$

$c = 20.2$

Find C:

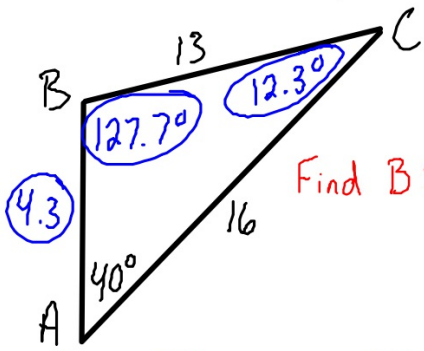
$\angle C = 180 - 52.3 - 40$

$\angle C = 87.7°$

Example

Solve △ ABC:

4B. ∠ A = 40°, a = 13, b = 16



$\frac{\sin 40}{13} = \frac{\sin B}{16} = \frac{\sin C}{c}$

$16 \sin 40 = 13 \sin B$

$\sin B = \frac{16 \sin 40}{13}$

Find B: $\angle B = \sin^{-1}\left(\frac{16 \sin 40}{13}\right)$

$\angle B \approx 52.3°$

Given two sides, one of the sides could swing in the opposite direction causing the triangle to have an obtuse angle making the measure of B we just found a reference angle to angle B.

This then would give angle B = 127.7°
Angle C then = 180° - 127.7° - 40° = 12.3°

$\frac{\sin 40}{13} = \frac{\sin 12.3}{c}$

$c \sin 40 = 13 \sin 12.3$

$c = \frac{13 \sin 12.3}{\sin 40} \approx 4.3$