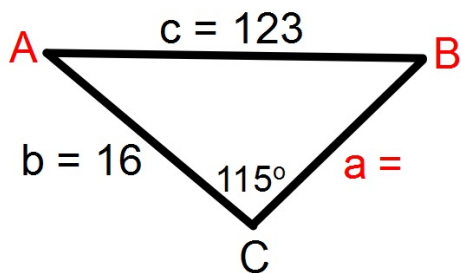


Bellwork

Solve the following triangle:

1. $\angle C = 115^\circ$, $b = 16$, & $c = 123$



First set up the Law of Sines:

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

$$\frac{\sin A}{a} = \frac{\sin B}{16} = \frac{\sin 115^\circ}{123}$$

After setting up the Law of Sines:

$$\frac{\sin B}{16} = \frac{\sin 115^\circ}{123}$$

$$\frac{123 \sin B}{123} = \frac{16 \sin 115^\circ}{123}$$

$$\sin B = \frac{16 \sin 115^\circ}{123}$$

$$\angle B = \sin^{-1} \left(\frac{16 \sin 115^\circ}{123} \right) \approx \boxed{6.8^\circ}$$

After finding B, find A:

$$\angle A = 180 - 115 - 6.8$$

$$\boxed{\angle A = 58.2^\circ}$$

Finally, find side a:

$$\frac{\sin 58.2^\circ}{a} = \frac{\sin 115^\circ}{123}$$

$$\frac{a \cdot \sin 115^\circ}{\sin 115^\circ} = \frac{123 \sin 58.2^\circ}{\sin 115^\circ}$$

$$\boxed{a = 115.3}$$

The Law of Cosines

There are 3 ways to set this up, but if you can remember one of them you should be able to remember them all:

$$1. a^2 = b^2 + c^2 - 2bc \cdot \cos A$$

$$2. b^2 = a^2 + c^2 - 2ac \cdot \cos B$$

$$3. c^2 = a^2 + b^2 - 2ab \cdot \cos C$$

When do I use the Law of Cosines?

Solving triangles when you are given:

1. 2 sides and the included angle.

This will assist in finding the 3rd side that has not been given.

2. All 3 of the sides.

This will assist in finding any of three missing angles.

Can you join this with the Law of Sines?

As with any math concepts, using multiple approaches is always encouraged if it will simplify the process.

Example

Solve the following triangle formed by the given information:

1. $a = 24$, $b = 32$, & $c = 37$

Set up one of the Law of Cosines:

$$\begin{aligned}a^2 &= b^2 + c^2 - 2bc \cdot \cos A \\24^2 &= 32^2 + 37^2 - 2(32)(37)\cos A \\576 &= 1024 + 1369 - 2368\cos A \\576 &= 2393 - 2368\cos A \\576 - 2393 &= -2368\cos A \\-1817 &= -2368\cos A \\ \frac{-1817}{-2368} &= \frac{-2368\cos A}{-2368} \\(1817/2368) &= \cos A \\ \cos^{-1}(1817/2368) &= A \\ \boxed{39.9^\circ \approx \angle A}\end{aligned}$$

Now use Law of Sines:

$$\frac{\sin 39.9}{24} = \frac{\sin B}{32} = \frac{\sin C}{37}$$

Finding B:

$$24 \sin B = 32 \sin 39.9$$
$$\frac{24}{24} \sin B = \frac{32 \sin 39.9}{24}$$

$$B = \sin^{-1}\left(\frac{32 \sin 39.9}{24}\right)$$

$$\boxed{\angle B \approx 58.8^\circ}$$

Finding C:

$$\angle C = 180 - 39.9 - 58.8$$

$$\boxed{\angle C = 81.3^\circ}$$

Example

Solve the following triangle formed by the given information:

2. $\angle C = 115^\circ$, $a = 16$, & $b = 52$

Set up one of the Law of Cosines:

Hint - Use the one that matches the angle given.

$$c^2 = a^2 + b^2 - 2ab \cos C$$

$$c^2 = 16^2 + 52^2 - 2(16)(52)\cos 115$$

$$c^2 = 256 + 2704 - 1664\cos 115$$

$$\sqrt{c^2} = \sqrt{2960 - 1664\cos 115}$$

$$c \approx 60.5$$

Set up the Law of Sines:

$$\frac{\sin A}{16} = \frac{\sin B}{52} = \frac{\sin 115}{60.5}$$

Let's find B:

$$\frac{60.5\sin B}{60.5} = \frac{52\sin 115}{60.5}$$

$$B = \sin^{-1}\left(\frac{52\sin 115}{60.5}\right)$$

$$\angle B \approx 51.2^\circ$$

Find C:

$$C = 180 - 51.2 - 115$$

$$\angle C = 13.8^\circ$$