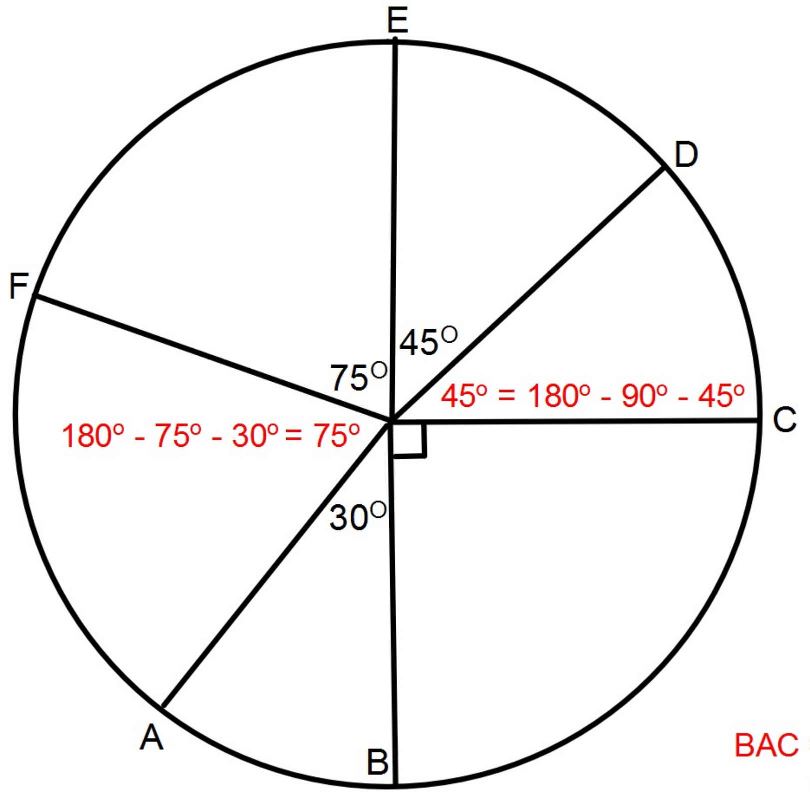
Bellwork

Find the measure of each of the following arcs:



1. $m\overset{\frown}{FD}$

   FD = FE + ED
   FD = 75° + 45°
   FD = 120°

2. $m\overset{\frown}{FCB}$

   FCB = FE + ED + DC + CB
   = 75° + 45° + 45° + 90° = **255°**

3. $m\overset{\frown}{BA}$

   BA = 30°

4. $m\overset{\frown}{FAC}$

   FAC = FA + AB + BC
   = 75° + 30° + 90° = **195°**

5. $m\overset{\frown}{BAC}$

   BAC = BA + AF + FE + ED + DC
   = 30° + 75° + 75° + 45° + 45° = **270°**

Angles of Focus

Throughout the unit there will be three main types of angles that we will focus on:

1. Central Angles

   We touched on this type of angle in the previous section and on the bellwork.

2. Inscribed Angles

   These will be the main focus of this lesson.

3. Circumscribed Angles

   This will be the focus of the next lesson.

Inscribed Angles

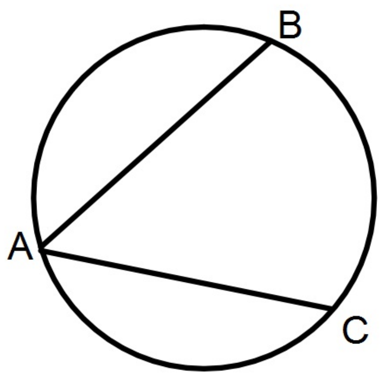
Inscribed Angle:

An angle whose vertex is on a circle and sides contain chords of that circle.

Intercepted Arc:

The arc between the angle on the circles outer edge.

$\angle$BAC is an inscribed angle

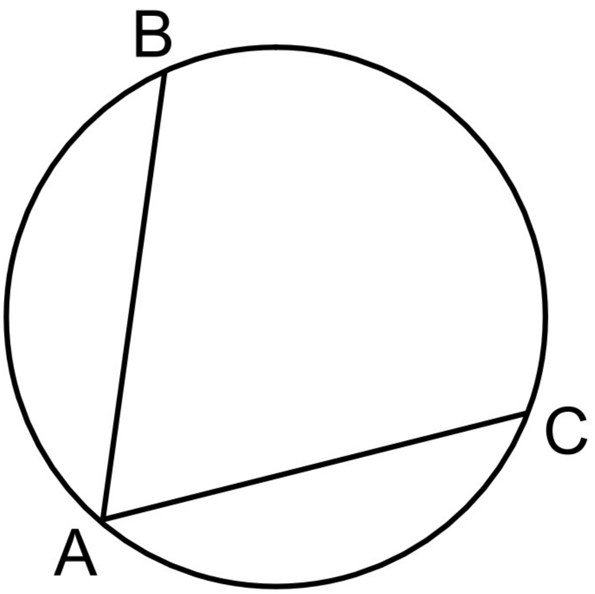


$\overset{\frown}{BC}$ is an intercepted arc

Measurement Theorem

Measure of an Inscribed Angle Theorem:

The measure of an inscribed angle is one half the measure of its intercepted arc.



$$\angle BAC = \frac{1}{2} \cdot \overset{\frown}{BC}$$

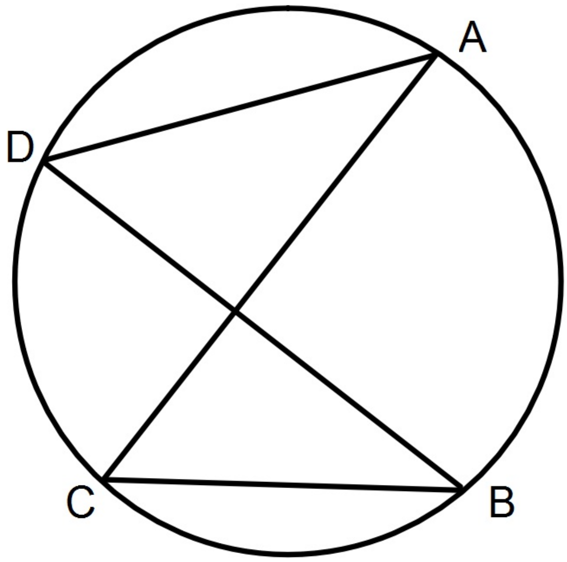
OR

$$2(\angle BAC) = \overset{\frown}{BC}$$

Another Inscribed Angle Theorem

Theorem:

If two inscribed angles of a circle intercept the same arc,
Then the angles are congruent.

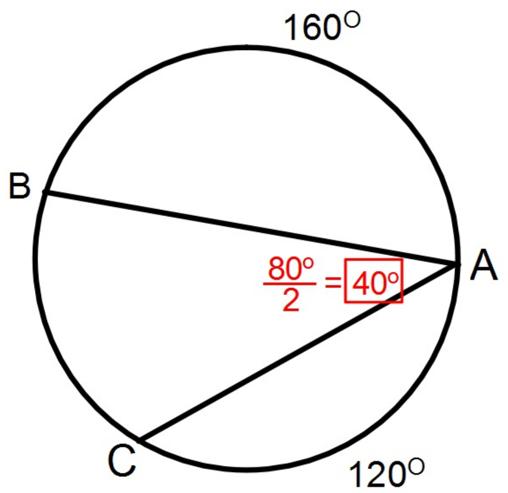


$\angle ADB \cong \angle ACB$

Examples

Find the indicated measure:

1. $m\angle A$

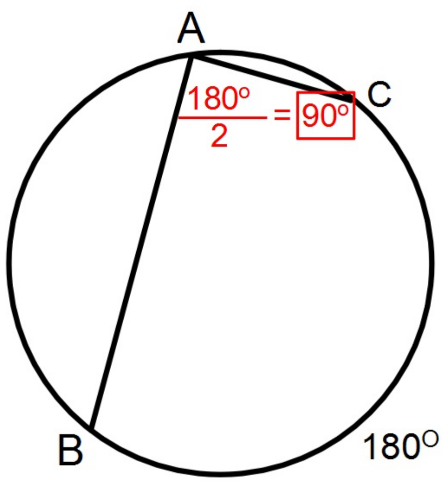


In order to find $\angle A$, we will need to find the measure of $\overset{\frown}{BC}$.

Since there is 360° in a circle...

$\overset{\frown}{BC} = 360° - 160° - 120° = 80°$

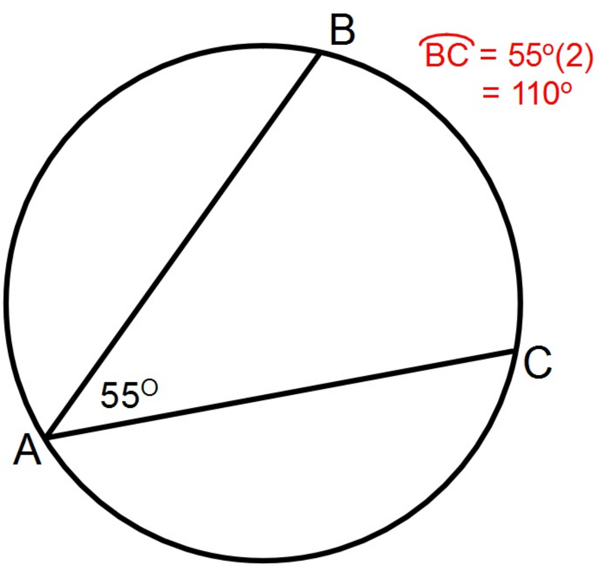
2. $m\angle A$



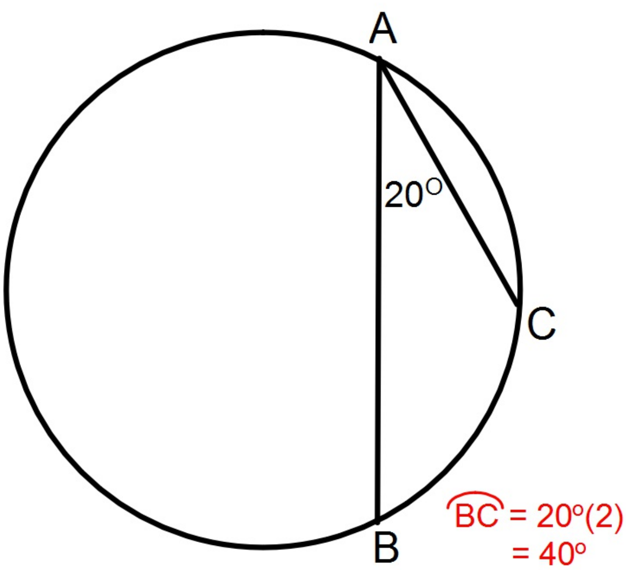
Examples

Find the indicated measure:

3. $m\overset{\frown}{BC}$



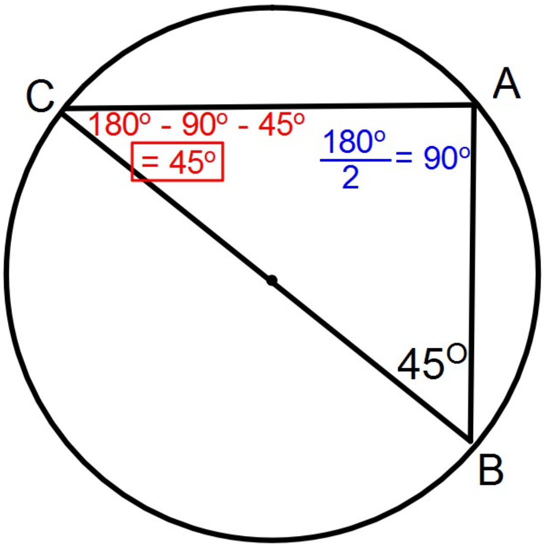
4. $m\overset{\frown}{BC}$



Examples

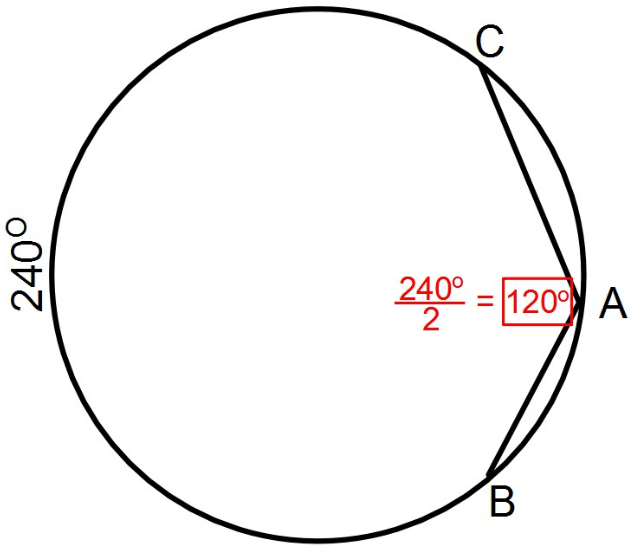
Find the indicated measure:

5. m∠C



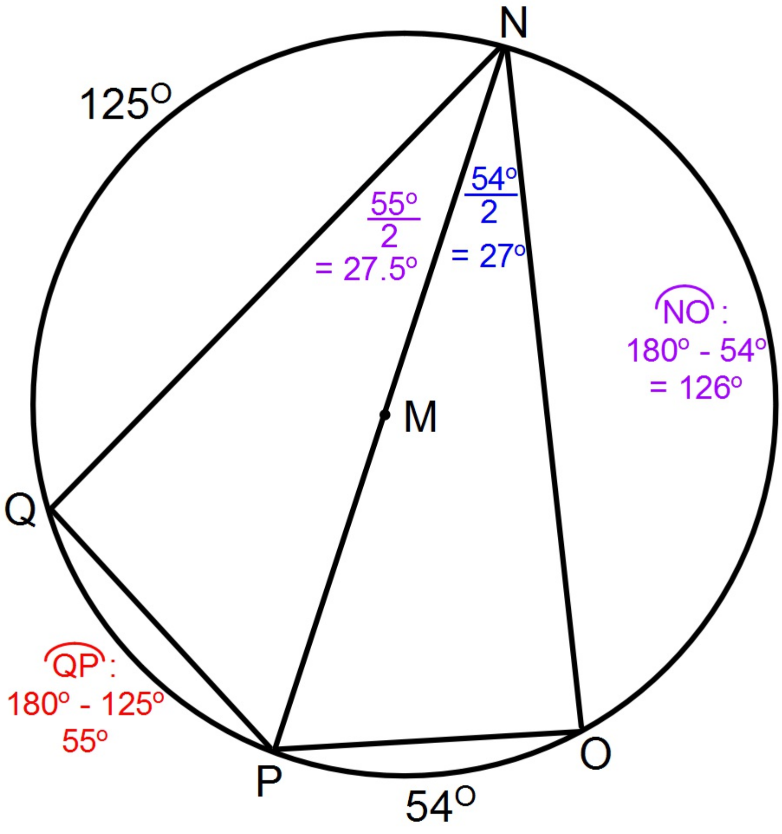
Since $\overline{CB}$ is the diameter of the circle we can say $\overset{\frown}{CB} = 180°$

6. m∠A



Examples

Find the indicated measure in ⊙M:



7. m∠PNO

As shown 27°

8. m$\overset{\frown}{QO}$

$\overset{\frown}{QO} = \overset{\frown}{QP} + \overset{\frown}{PO}$

$\overset{\frown}{QO} = 55^\circ + 54^\circ = 109^\circ$

9. m$\overset{\frown}{OQN}$

$\overset{\frown}{OQN} = \overset{\frown}{NQ} + \overset{\frown}{QO}$

$\overset{\frown}{OQN} = 125^\circ + 109^\circ = 234^\circ$

10. m∠QNP

As shown 27.5°

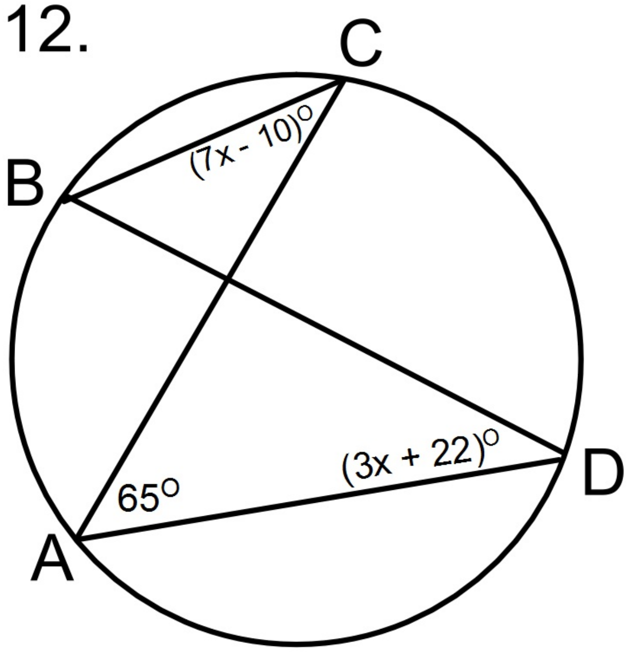
11. m$\overset{\frown}{PQ}$

As shown 55°

Examples

Find m∠B and m∠C:

12.



$\overset{\frown}{BA}$ is a shared Intercepted Arc for ∠C and ∠D.

This means that ∠C = ∠D.

So...

$$7x - 10 = 3x + 22$$
$$-3x \qquad -3x$$
$$4x - 10 = 22$$
$$+10 \quad +10$$
$$\frac{4x}{4} = \frac{32}{4}$$
$$x = 8$$

This means that $\angle C = (7(8) - 10)^o$

$\angle C = (56 - 10)^o$

$\boxed{\angle C = 46^o}$

$\overset{\frown}{CD}$ is a shared Intercepted Arc for ∠A and ∠D. This means that since $\angle A = 65^o$ so is ∠B. Therefore, $\boxed{\angle B = 65^o}$