

Summations – Day 3
Unit 6: Representations of Linear Relations

Evaluate the related series of each sequence.

<p>1. 30, 39, 48, 57, 66, 75, 84</p> $30 + 39 + 48 + 57 + 66 + 75 + 84 = \boxed{399}$	<p>2. 22, 29, 36, 43, 50, 57, 64</p> $22 + 29 + 36 + 43 + 50 + 57 + 64 = \boxed{301}$
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Evaluate each arithmetic series described.

<p>3. $46 + 56 + 66 + 76 \dots, n = 9$</p> $S_9 = \boxed{774}$	<p>4. $(-10) + (-14) + (-18) + (-22) \dots, n = 7$</p> $S_7 = \boxed{-154}$
<p>5. $20 + 28 + 36 + 44 \dots, n = 15$</p> $S_{15} = \boxed{1140}$	<p>6. $6 + 10 + 14 + 18 \dots, n = 10$</p> $S_{10} = \boxed{240}$
<p>7. $\sum_{i=1}^{10} (10 - 9i)$</p> <p>$d = -9$ $n = 10$ $a_i = 10 - 9(1)$ $a_i = 10 - 9$ $a_i = 1$</p> $S_{10} = \frac{n}{2} (2 \cdot a_1 + (n-1) \cdot d)$ $S_{10} = \frac{10}{2} (2 \cdot 1 + (10-1) \cdot (-9))$ $S_{10} = 5(2 + (9)(-9))$ $S_{10} = 5(2 - 81)$ $S_{10} = 5(-79)$ $S_{10} = \boxed{-395}$	<p>8. $\sum_{k=1}^9 (3k - 6)$</p> <p>$d = 3$ $n = 9$ $a_i = 3(1) - 6$ $a_i = 3 - 6$ $a_i = -3$</p> $S_9 = \frac{n}{2} (2 \cdot a_1 + (n-1) \cdot d)$ $S_9 = \frac{9}{2} (2 \cdot (-3) + (9-1) \cdot 3)$ $S_9 = 4.5(-6 + (8)(3))$ $S_9 = 4.5(-6 + 24)$ $S_9 = 4.5(18)$ $S_9 = \boxed{81}$
<p>9. $\sum_{i=1}^7 (16 - 10i)$</p> <p>$d = -10$ $n = 7$ $a_i = 16 - 10(1)$ $a_i = 16 - 10$ $a_i = 6$</p> $S_7 = \frac{n}{2} (2 \cdot a_1 + (n-1) \cdot d)$ $S_7 = \frac{7}{2} (2 \cdot 6 + (7-1) \cdot (-10))$ $S_7 = 3.5(12 + (6)(-10))$ $S_7 = 3.5(12 - 60)$ $S_7 = 3.5(-48)$ $S_7 = \boxed{-168}$	<p>10. $\sum_{i=1}^5 (10i - 14)$</p> <p>$d = 10$ $n = 5$ $a_i = 10(1) - 14$ $a_i = 10 - 14$ $a_i = -4$</p> $S_5 = \frac{n}{2} (2 \cdot a_1 + (n-1) \cdot d)$ $S_5 = \frac{5}{2} (2 \cdot (-4) + (5-1) \cdot 10)$ $S_5 = 2.5(-8 + (4)(10))$ $S_5 = 2.5(-8 + 40)$ $S_5 = 2.5(32)$ $S_5 = \boxed{80}$

Evaluate each arithmetic series described.

11. $a_1 = 7, a_n = 91, n = 15$

$$S_{15} = \frac{n}{2} (a_1 + a_n)$$

$$S_{15} = \frac{15}{2} (7 + 91)$$

$$S_{15} = 7.5 (98)$$

$$S_{15} = 735$$

12. $a_1 = 22, a_n = 58, n = 10$

$$S_{10} = \frac{n}{2} (a_1 + a_n)$$

$$S_{10} = \frac{10}{2} (22 + 58)$$

$$S_{10} = 5 (80)$$

$$S_{10} = 400$$

13. $a_1 = 2, a_n = 22, n = 6$

$$S_6 = \frac{n}{2} (a_1 + a_n)$$

$$S_6 = \frac{6}{2} (2 + 22)$$

$$S_6 = 3 (24)$$

$$S_6 = 72$$

14. $a_1 = 3, a_n = 63, n = 11$

$$S_{11} = \frac{n}{2} (a_1 + a_n)$$

$$S_{11} = \frac{11}{2} (3 + 63)$$

$$S_{11} = 5.5 (66)$$

$$S_{11} = 363$$

15. $a_1 = 33, a_n = 93, n = 7$

$$S_7 = \frac{n}{2} (a_1 + a_n)$$

$$S_7 = \frac{7}{2} (33 + 93)$$

$$S_7 = 3.5 (126)$$

$$S_7 = 441$$

16. $a_1 = -25, a_n = -89, n = 9$

$$S_9 = \frac{n}{2} (a_1 + a_n)$$

$$S_9 = \frac{9}{2} (-25 - 89)$$

$$S_9 = 4.5 (-114)$$

$$S_9 = -513$$

17. $a_1 = 20, d = 9, n = 10$

$$S_{10} = \frac{n}{2} (2 \cdot a_1 + (n-1) \cdot d)$$

$$S_{10} = \frac{10}{2} (2 \cdot 20 + (10-1) \cdot 9)$$

$$S_{10} = 5 (40 + (9)(9))$$

$$S_{10} = 5 (40 + 81)$$

$$S_{10} = 5 (121) = 605$$

18. $a_1 = 17, d = 2, n = 12$

$$S_{12} = \frac{n}{2} (2 \cdot a_1 + (n-1) \cdot d)$$

$$S_{12} = \frac{12}{2} (2 \cdot 17 + (12-1) \cdot 2)$$

$$S_{12} = 6 (34 + (11)(2))$$

$$S_{12} = 6 (34 + 22)$$

$$S_{12} = 6 (56) = 336$$

19. $a_1 = 29, d = 7, n = 15$

$$S_{15} = \frac{n}{2} (2 \cdot a_1 + (n-1) \cdot d)$$

$$S_{15} = \frac{15}{2} (2 \cdot 29 + (15-1) \cdot 7)$$

$$S_{15} = 7.5 (58 + (14)(7))$$

$$S_{15} = 7.5 (58 + 98)$$

$$S_{15} = 7.5 (156) = 1170$$

20. $a_1 = 2, d = 3, n = 13$

$$S_{13} = \frac{n}{2} (2 \cdot a_1 + (n-1) \cdot d)$$

$$S_{13} = \frac{13}{2} (2 \cdot 2 + (13-1) \cdot 3)$$

$$S_{13} = 6.5 (4 + (12)(3))$$

$$S_{13} = 6.5 (4 + 36)$$

$$S_{13} = 6.5 (40) = 260$$

③ Information Needed

$$d = 56 - (46) = 10$$

$$a_1 = 46$$

$$n = 9$$

Using the formula

$$S_n = \frac{n}{2} (2 \cdot a_1 + (n-1) \cdot d)$$

$$S_n = \frac{9}{2} (2 \cdot 46 + (9-1) \cdot 10)$$

$$S_n = 4.5 (92 + (8)(10))$$

$$S_n = 4.5 (92 + 80)$$

$$S_n = 4.5 (172)$$

$$S_n = 774$$

④ Information Needed

$$d = -14 - (-10) = -4$$

$$a_1 = -10$$

$$n = 7$$

Using the formula

$$S_n = \frac{n}{2} (2 \cdot a_1 + (n-1) \cdot d)$$

$$S_n = \frac{7}{2} (2 \cdot -10 + (7-1) \cdot -4)$$

$$S_n = 3.5 (-20 + (6)(-4))$$

$$S_n = 3.5 (-20 - 24)$$

$$S_n = 3.5 (-44)$$

$$S_n = -154$$

⑤ Information Needed

$$d = 28 - (20) = 8$$

$$a_1 = 20$$

$$n = 15$$

Using the formula

$$S_n = \frac{n}{2} (2 \cdot a_1 + (n-1) \cdot d)$$

$$S_n = \frac{15}{2} (2 \cdot 20 + (15-1) \cdot 8)$$

$$S_n = 7.5 (40 + (14)(8))$$

$$S_n = 7.5 (40 + 112)$$

$$S_n = 7.5 (152)$$

$$S_n = 1140$$

⑥ Information Needed

$$d = 10 - (6) = 4$$

$$a_1 = 6$$

$$n = 10$$

Using the formula

$$S_n = \frac{n}{2} (2 \cdot a_1 + (n-1) \cdot d)$$

$$S_n = \frac{10}{2} (2 \cdot 6 + (10-1) \cdot 4)$$

$$S_n = 5 (12 + (9)(4))$$

$$S_n = 5 (12 + 36)$$

$$S_n = 5 (48)$$

$$S_n = 240$$

Using the formula

$$(a_1 + a_n) \cdot \frac{n}{2} = S_n$$

$$(6 + a_{10}) \cdot \frac{10}{2} = 240$$

$$(6 + a_{10}) \cdot 5 = 240$$

$$6 + a_{10} = \frac{240}{5}$$

$$6 + a_{10} = 48$$

$$a_{10} = 48 - 6$$

$$a_{10} = 42$$

Using the formula

$$S_n = \frac{n}{2} (2a_1 + (n-1)d)$$

$$240 = \frac{10}{2} (2(6) + (10-1)d)$$

$$240 = 5 (12 + 9d)$$

$$240 = 60 + 45d$$

$$180 = 45d$$

$$d = \frac{180}{45}$$

$$d = 4$$

Using the formula

$$S_n = \frac{n}{2} (2a_1 + (n-1)d)$$

$$240 = \frac{10}{2} (2(6) + (10-1)d)$$

$$240 = 5 (12 + 9d)$$

$$240 = 60 + 45d$$

$$180 = 45d$$

$$d = \frac{180}{45}$$

$$d = 4$$

Using the formula

$$S_n = \frac{n}{2} (2a_1 + (n-1)d)$$

$$240 = \frac{10}{2} (2(6) + (10-1)d)$$

$$240 = 5 (12 + 9d)$$

$$240 = 60 + 45d$$

$$180 = 45d$$

$$d = \frac{180}{45}$$

$$d = 4$$