Summations – Day 2

Unit 7: Representations of Exponential Relations

Evaluate the related series of each sequence.

1. $-3, -6, -12, -24$ $-3-6-12-24 = \boxed{-45}$	2. $-4, 12, -36, 108$ $-4+12-36+108 = \boxed{80}$

Evaluate each geometric series described.

3. $2 - 12 + 72 - 432 \ldots, n = 6$ $S_6 = -13330$	4. $1 - 5 + 25 - 125 \ldots, n = 7$ $S_7 = 13021$
5. $\sum_{m=1}^{10} 4^{m-1}$ $S_{10} = 349525$	6. $\sum_{n=1}^{9} 4^{n-1}$ $S_9 = 87381$
7. $\sum_{k=1}^{7} (-6)^{k-1}$ $S_7 = 39991$	8. $\sum_{k=1}^{10} -2^{k-1}$ $S_{10} = -1023$
9. $a_1 = -4, a_n = -186624, r = 6$ $S_n = -223948$	10. $a_1 = -2, a_n = -32768, r = 4$ $S_n = -43690$

Evaluate each geometric series described.

11. $a_1 = -4, a_n = -262144, r = -4$ $S_n = -209716$	12. $a_1 = 2, a_n = 524288, r = 4$ $S_n = 699050$
13. $a_1 = 2, a_7 = 1458, r = 3$ $S_7 = 2186$	14. $a_1 = 2, a_{10} = 39366, r = 3$ $S_{10} = 59048$
15. $a_1 = 2, a_7 = 93312, r = 6$ $S_7 = 111974$	16. $a_1 = 1, a_7 = 4096, r = 4$ $S_7 = 5461$
17. $a_1 = 4, r = 3, n = 8$ $S_8 = 13120$	18. $a_1 = -4, r = -2, n = 8$ $S_8 = 340$
19. $a_1 = -4, r = 4, n = 8$ $S_8 = -87380$	20. $a_1 = 1, r = -3, n = 10$ $S_{10} = -14762$

③ $2 - 12 + 72 - 432\ldots,\ n = 6$

$a_1 = 2,\ n = 6,\ r = \frac{-12}{2} = -6$

$$S_n = \frac{a_1(1-r^n)}{1-r}$$

$$S_6 = \frac{2(1-(-6)^6)}{1-(-6)} = \frac{2(1-46656)}{1-(-6)} = \frac{2(-46655)}{1+6} = \frac{-93310}{7} = \boxed{-13330}$$

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④ $1 - 5 + 25 - 125\ldots,\ n = 7$

$a_1 = 1,\ n = 7,\ r = \frac{-5}{1} = -5$

$$S_n = \frac{a_1(1-r^n)}{1-r}$$

$$S_7 = \frac{1(1-(-5)^7)}{1-(-5)} = \frac{1(1-(-78125))}{1-(-5)} = \frac{1(1+78125)}{1+5} = \frac{1(78126)}{6} = \boxed{13021}$$

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⑤ $\sum_{m=1}^{10} 4^{m-1}$

$a_1 = 1,\ n = 10,\ r = 4$

$$S_n = \frac{a_1(1-r^n)}{1-r}$$

$$S_{10} = \frac{1(1-4^{10})}{1-4} = \frac{1(1-1048576)}{1-4} = \frac{1(-1048575)}{-3} = \frac{-1048575}{-3} = \boxed{349525}$$

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⑥ $\sum_{n=1}^{9} 4^{n-1}$

$a_1 = 1,\ n = 9,\ r = 4$

$$S_n = \frac{a_1(1-r^n)}{1-r}$$

$$S_9 = \frac{1(1-4^9)}{1-4} = \frac{1(1-262144)}{1-4} = \frac{1(-262143)}{-3} = \frac{-262143}{-3} = \boxed{87381}$$

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⑦ $\sum_{K=1}^{7} (-6)^{K-1}$

$a_1 = 1,\ n = 7,\ r = -6$

$$S_n = \frac{a_1(1-r^n)}{1-r}$$

$$S_7 = \frac{1(1-(-6)^7)}{1-(-6)} = \frac{1(1-(-279936))}{1-(-6)} = \frac{1(1+279936)}{1-(-6)} = \frac{1(279937)}{7} = \boxed{39991}$$

(8) $\sum_{k=1}^{10} -2^{k-1}$ $\quad a_1 = -1, \quad n = 10, \quad r = 2$

$$S_n = \frac{a_1(1-r^n)}{1-r}$$

$$S_{10} = \frac{-1(1-2^{10})}{1-2} = \frac{-1(1-1024)}{1-2} = \frac{-1(-1023)}{-1} = \boxed{-1023}$$

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(9) $a_1 = -4, \quad a_n = -186624, \quad r = 6$

$$S_n = \frac{a_1 - a_n \cdot r}{1-r}$$

$$S_n = \frac{-4-(-186624)(6)}{1-6} = \frac{-4-(-1119744)}{1-6} = \frac{-4+1119744}{-5} = \frac{1119740}{-5} = \boxed{-223948}$$

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(10) $a_1 = -2, \quad a_n = -32768, \quad r = 4$

$$S_n = \frac{a_1 - a_n \cdot r}{1-r}$$

$$S_n = \frac{-2-(-32768)(4)}{1-4} = \frac{-2-(-131072)}{1-4} = \frac{-2+131072}{1-4} = \frac{131070}{-3} = \boxed{-43690}$$

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(11) $a_1 = -4, \quad a_n = -262144, \quad r = -4$

$$S_n = \frac{a_1 - a_n \cdot r}{1-r}$$

$$S_n = \frac{-4-(-262144)(-4)}{1-(-4)} = \frac{-4-1048576}{1-(-4)} = \frac{-1048580}{1+4} = \frac{-1048580}{5} = \boxed{-209716}$$

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(12) $a_1 = 2, \quad a_n = 524288, \quad r = 4$

$$S_n = \frac{a_1 - a_n \cdot r}{1-r}$$

$$S_n = \frac{2-524288(4)}{1-4} = \frac{2-2097152}{1-4} = \frac{-2097150}{-3} = \boxed{699050}$$

(13) $a_1 = 2, \quad a_7 = 1458, \quad r = 3$

$$S_n = \frac{a_1 - a_n \cdot r}{1 - r}$$

$$S_n = \frac{2 - 1458(3)}{1 - 3} = \frac{2 - 4374}{1 - 3} = \frac{-4372}{-2} = \boxed{2186}$$

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(14) $a_1 = 2, \quad a_{10} = 39366, \quad r = 3$

$$S_n = \frac{a_1 - a_n \cdot r}{1 - r}$$

$$S_n = \frac{2 - 39366(3)}{1 - 3} = \frac{2 - 118098}{1 - 3} = \frac{-118096}{-2} = \boxed{59048}$$

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(15) $a_1 = 2, \quad a_7 = 93312, \quad r = 6$

$$S_n = \frac{a_1 - a_n \cdot r}{1 - r}$$

$$S_n = \frac{2 - 93312(6)}{1 - 6} = \frac{2 - 559872}{1 - 6} = \frac{-559870}{-5} = \boxed{111974}$$

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(16) $a_1 = 1, \quad a_7 = 4096, \quad r = 4$

$$S_n = \frac{a_1 - a_n \cdot r}{1 - r}$$

$$S_n = \frac{1 - 4096(4)}{1 - 4} = \frac{1 - 16384}{1 - 4} = \frac{-16383}{-3} = \boxed{5461}$$

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(17) $a_1 = 4, \quad r = 3, \quad n = 8$

$$S_n = \frac{a_1 (1 - r^n)}{1 - r}$$

$$S_8 = \frac{4(1 - 3^8)}{1 - 3} = \frac{4(1 - 6561)}{1 - 3} = \frac{4(-6560)}{-2} = \frac{-26240}{-2} = \boxed{13120}$$

(18) $a_1 = -4, \quad r = -2, \quad n = 8$

$$S_n = \frac{a_1(1-r^n)}{1-r}$$

$$S_8 = \frac{-4(1-(-2)^8)}{1-(-2)} = \frac{-4(1-256)}{1-(-2)} = \frac{-4(-255)}{1+2} = \frac{1020}{3} = \boxed{340}$$

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(19) $a_1 = -4, \quad r = 4, \quad n = 8$

$$S_n = \frac{a_1(1-r^n)}{1-r}$$

$$S_8 = \frac{-4(1-4^8)}{1-4} = \frac{-4(1-65536)}{1-4} = \frac{-4(-65535)}{-3} = \frac{262140}{-3} = \boxed{-87380}$$

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(20) $a_1 = 1, \quad r = -3, \quad n = 10$

$$S_n = \frac{a_1(1-r^n)}{1-r}$$

$$S_{10} = \frac{1(1-(-3)^{10})}{1-(-3)} = \frac{1(1-59049)}{1-(-3)} = \frac{1(-59048)}{1+3} = \frac{-59048}{4} = \boxed{-14762}$$