Unit 7: Representations of Exponential Relations

POST TEST – Version A

Determine if the sequence is geometric. If it is, find the common ratio.

1. 2, 10, 50, 250 ...

$\frac{10}{2} = 5$ $\frac{50}{10} = 5$ $\frac{250}{50} = 5$

Geometric $r = 5$

2. 4, 16, 64, 256, ...

$\frac{16}{4} = 4$ $\frac{64}{16} = 4$ $\frac{256}{64} = 4$

Geometric $r = 4$

Find the recursive formula for each of the following:

3. 2, 12, 72, 432, ...

$\frac{12}{2} = 6$ $\frac{72}{12} = 6$ $\frac{432}{72} = 6$

$a_1 = 2$ $a_n = a_{n-1} \cdot 6$

4. $-2, -12, -72, -432, \ldots$

$\frac{-12}{-2} = 6$ $\frac{-72}{-12} = 6$ $\frac{-432}{-72} = 6$

$a_1 = -2$ $a_n = a_{n-1} \cdot 6$

Find the explicit formula for each of the following:

5. $-2, -4, -8, -16, \ldots$

$\frac{-4}{-2} = 2$ $\frac{-8}{-4} = 2$ $\frac{-16}{-8} = 2$

$a_n = a_1 \cdot r^{n-1}$ $a_n = -2(2)^{n-1}$

6. 4, 24, 144, 864, ...

$\frac{24}{4} = 6$ $\frac{144}{24} = 6$ $\frac{864}{144} = 6$

$a_n = a_1 \cdot r^{n-1}$ $a_n = 4(6)^{n-1}$

Given the following geometric sequences answer each of the following:

7. $-3, -12, -48, -192, \ldots$

A. Find the next three terms

$\frac{-12}{-3} = 4$ $\frac{-48}{-12} = 4$ $\frac{-192}{-48} = 4$

$a_5 = -192 \cdot 4 = -768$

$a_6 = -768 \cdot 4 = -3072$

$a_7 = -3072 \cdot 4 = -12288$

B. Find a_8

Explicit Formula

$a_n = -3(4)^{n-1}$

$a_8 = -12288 \cdot 4$

$a_8 = -49152$

C. Find a_9

$a_9 = -49152 \cdot 4$

$a_9 = -196608$

8. 1, 2, 4, 8, ...

A. Find the next three terms

$\frac{2}{1} = 2$ $\frac{4}{2} = 2$ $\frac{8}{4} = 2$

$a_5 = 8 \cdot 2 = 16$

$a_6 = 16 \cdot 2 = 32$

$a_7 = 32 \cdot 2 = 64$

B. Find a_8

Explicit Formula

$a_n = 1(2)^{n-1}$

$a_8 = 64 \cdot 2 = 128$

C. Find a_{10}

$a_9 = 128 \cdot 2 = 256$

$a_{10} = 256 \cdot 2 = 512$

Find the missing term or terms in each geometric sequence.

9. …, −3, ___, −75, … $a_2 = -15$	10. …, −1, ___, −4, … $a_2 = -2$
11. …, 3, ___, ___, 375, … $a_2 = 15$ $a_3 = 75$	12. …, −2, ___, ___, −54, … $a_2 = -6$ $a_3 = -18$
13. …, 3, ___, ___, ___, 3888, … $a_2 = 18$ $a_3 = 108$ $a_4 = 648$	14. …, 3, ___, ___, ___, 1875, … $a_2 = 15$ $a_3 = 75$ $a_4 = 375$
15. …, 2, ___, ___, ___, ___, 2048, … $a_2 = 8$ $a_4 = 128$ $a_3 = 32$ $a_5 = 512$	16. …, 4, ___, ___, ___, ___, 972, … $a_2 = 12$ $a_4 = 108$ $a_3 = 36$ $a_5 = 324$
17. …, 2, ___, ___, ___, ___, ___, 1458, … $a_2 = 6$ $a_4 = 54$ $a_6 = 486$ $a_3 = 18$ $a_5 = 162$	18. …, −4, ___, ___, ___, ___, ___, −256, … $a_2 = -8$ $a_4 = -32$ $a_6 = -128$ $a_3 = -16$ $a_5 = -64$

Evaluate each geometric series described.

19. $2 - 6 + 18 - 54 \ldots, n = 8$ $S_8 = -3280$	20. $3 + 6 + 12 + 24 \ldots, n = 6$ $S_6 = 189$
21. $3 + 18 + 108 + 648 \ldots, n = 7$ $S_7 = 167961$	22. $-2 + 8 - 32 + 128 \ldots, n = 7$ $S_7 = -6554$
23. $\sum_{n=1}^{9} 4^{n-1}$ $S_9 = 87381$	24. $\sum_{i=1}^{9} 2 \cdot 2^{i-1}$ $S_9 = 1022$
25. $a_1 = -2, r = -2, n = 9$ $S_9 = -342$	26. $a_1 = 2, r = -4, n = 7$ $S_7 = 6554$

(9) $\ldots, \underset{a_1}{-3}, \underset{a_2}{\_\_}, \underset{a_3}{-75}, \ldots$

$a_2 = -3 \cdot 5 = \boxed{-15}$

$a_n = a_1 \cdot r^{n-1}$

$-75 = -3(r)^{3-1}$

$\frac{-75}{-3} = \frac{-3(r)^2}{-3}$

$\sqrt{25} = \sqrt{r^2}$

$5 = r$

---

(10) $\ldots, \underset{a_1}{-1}, \underset{a_2}{\_\_}, \underset{a_3}{-4}, \ldots$

$a_2 = -1 \cdot 2 = \boxed{-2}$

$a_n = a_1 \cdot r^{n-1}$

$-4 = -1(r)^{3-1}$

$\frac{-4}{-1} = \frac{-1(r)^2}{-1}$

$\sqrt{4} = \sqrt{r^2}$

$2 = r$

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(11) $\ldots, \underset{a_1}{3}, \underset{a_2}{\_\_}, \underset{a_3}{\_\_}, \underset{a_4}{375}, \ldots$

$a_2 = 3 \cdot 5 = \boxed{15}$

$a_3 = 15 \cdot 5 = \boxed{75}$

$a_n = a_1 \cdot r^{n-1}$

$375 = 3(r)^{4-1}$

$\frac{375}{3} = \frac{3(r)^3}{3}$

$\sqrt[3]{125} = \sqrt[3]{r^3}$

$5 = r$

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(12) $\ldots, \underset{a_1}{-2}, \underset{a_2}{\_\_}, \underset{a_3}{\_\_}, \underset{a_4}{-54}, \ldots$

$a_2 = -2 \cdot 3 = \boxed{-6}$

$a_3 = -6 \cdot 3 = \boxed{-18}$

$a_n = a_1 \cdot r^{n-1}$

$-54 = -2(r)^{4-1}$

$\frac{-54}{-2} = \frac{-2(r)^3}{-2}$

$\sqrt[3]{27} = \sqrt[3]{r^3}$

$3 = r$

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(13) $\ldots, 3, \_\_, \_\_, \_\_, 3888, \ldots$

$a_2 = 3 \cdot 6 = \boxed{18}$

$a_3 = 18 \cdot 6 = \boxed{108}$

$a_4 = 108 \cdot 6 = \boxed{648}$

$a_n = a_1 \cdot r^{n-1}$

$3888 = 3(r)^{5-1}$

$\frac{3888}{3} = \frac{3(r)^4}{3}$

$\sqrt[4]{1296} = \sqrt[4]{r^4}$

$6 = r$

(14) $\ldots, \underset{a_1}{3}, \underset{a_2}{\_\_}, \underset{a_3}{\_\_}, \underset{a_4}{\_\_}, \underset{a_5}{1875}, \ldots$

$a_2 = 3 \cdot 5 = \boxed{15}$

$a_3 = 15 \cdot 5 = \boxed{75}$

$a_4 = 75 \cdot 5 = \boxed{375}$

$a_n = a_1 \cdot r^{n-1}$

$1875 = 3(r)^{5-1}$

$\frac{1875}{3} = \frac{3(r)^4}{3}$

$\sqrt[4]{625} = \sqrt[4]{r^4}$

$5 = r$

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(15) $\ldots, \underset{a_1}{2}, \underset{a_2}{\_\_}, \underset{a_3}{\_\_}, \underset{a_4}{\_\_}, \underset{a_5}{\_\_}, \underset{a_6}{2048}, \ldots$

$a_2 = 2 \cdot 4 = \boxed{8}$

$a_3 = 8 \cdot 4 = \boxed{32}$

$a_4 = 32 \cdot 4 = \boxed{128}$

$a_5 = 128 \cdot 4 = \boxed{512}$

$a_n = a_1 \cdot r^{n-1}$

$2048 = 2(r)^{6-1}$

$\frac{2048}{2} = \frac{2(r)^5}{2}$

$\sqrt[5]{1024} = \sqrt[5]{r^5}$

$4 = r$

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(16) $\ldots, \underset{a_1}{4}, \underset{a_2}{\_\_}, \underset{a_3}{\_\_}, \underset{a_4}{\_\_}, \underset{a_5}{\_\_}, \underset{a_6}{972}, \ldots$

$a_2 = 4 \cdot 3 = \boxed{12}$

$a_3 = 12 \cdot 3 = \boxed{36}$

$a_4 = 36 \cdot 3 = \boxed{108}$

$a_5 = 108 \cdot 3 = \boxed{324}$

$a_n = a_1 \cdot r^{n-1}$

$972 = 4(r)^{6-1}$

$\frac{972}{4} = \frac{4(r)^5}{4}$

$\sqrt[5]{243} = \sqrt[5]{r^5}$

$3 = r$

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(17) $\ldots, \underset{a_1}{2}, \underset{a_2}{\_\_}, \underset{a_3}{\_\_}, \underset{a_4}{\_\_}, \underset{a_5}{\_\_}, \underset{a_6}{\_\_}, \underset{a_7}{1458}, \ldots$

$a_2 = 2 \cdot 3 = \boxed{6}$

$a_3 = 6 \cdot 3 = \boxed{18}$

$a_4 = 18 \cdot 3 = \boxed{54}$

$a_5 = 54 \cdot 3 = \boxed{162}$

$a_6 = 162 \cdot 3 = \boxed{486}$

$a_n = a_1 \cdot r^{n-1}$

$1458 = 2(r)^{7-1}$

$\frac{1458}{2} = \frac{2(r)^6}{2}$

$\sqrt[6]{729} = \sqrt[6]{r^6}$

$3 = r$

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(18) $\ldots, \underset{a_1}{-4}, \underset{a_2}{\_\_}, \underset{a_3}{\_\_}, \underset{a_4}{\_\_}, \underset{a_5}{\_\_}, \underset{a_6}{\_\_}, \underset{a_7}{-256}, \ldots$

$a_2 = -4 \cdot 2 = \boxed{-8}$

$a_3 = -8 \cdot 2 = \boxed{-16}$

$a_4 = -16 \cdot 2 = \boxed{-32}$

$a_5 = -32 \cdot 2 = \boxed{-64}$

$a_6 = -64 \cdot 2 = \boxed{-128}$

$a_n = a_1 \cdot r^{n-1}$

$-256 = -4(r)^{7-1}$

$\frac{-256}{-4} = \frac{-4(r)^6}{-4}$

$\sqrt[6]{64} = \sqrt[6]{r^6}$

$2 = r$

(19) $2-6+18-54\ldots,\ n=8$ $\quad a_1=2;\quad n=8;\quad r=\frac{-6}{2}=-3$

$$S_n=\frac{a_1(1-r^n)}{1-r}$$

$$S_8=\frac{2(1-(-3)^8)}{1-(-3)}=\frac{2(1-6561)}{1-(-3)}=\frac{2(-6560)}{1+3}=\frac{-13120}{4}=\boxed{-3280}$$

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(20) $3+6+12+24\ldots,\ n=6$ $\quad a_1=3;\quad n=6;\quad r=\frac{6}{3}=2$

$$S_n=\frac{a_1(1-r^n)}{1-r}$$

$$S_6=\frac{3(1-2^6)}{1-2}=\frac{3(1-64)}{1-2}=\frac{3(-63)}{-1}=\frac{-189}{-1}=\boxed{189}$$

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(21) $3+18+108+648\ldots,\ n=7$ $\quad a_1=3;\quad n=7;\quad r=\frac{18}{3}=6$

$$S_n=\frac{a_1(1-r^n)}{1-r}$$

$$S_7=\frac{3(1-6^7)}{1-6}=\frac{3(1-279936)}{1-6}=\frac{3(-279935)}{-5}=\frac{-839805}{-5}$$

$$\boxed{S_7=167961}$$

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(22) $-2+8-32+128\ldots,\ n=7$ $\quad a_1=-2;\quad n=7;\quad r=\frac{8}{-2}=-4$

$$S_n=\frac{a_1(1-r^n)}{1-r}$$

$$S_7=\frac{-2(1-(-4)^7)}{1-(-4)}=\frac{-2(1-(-16384))}{1-(-4)}=\frac{-2(1+16384)}{1+4}=\frac{-2(16385)}{5}$$

$$S_7=\frac{-32770}{5}=\boxed{-6554}$$

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(23) $\sum_{n=1}^{9} 4^{n-1}$ $\quad a_1=1;\quad n=9;\quad r=4$

$$S_n=\frac{a_1(1-r^n)}{1-r}$$

$$S_9=\frac{1(1-4^9)}{1-4}=\frac{1(1-262144)}{1-4}=\frac{1(-262143)}{-3}=\frac{-262143}{-3}=\boxed{87381}$$

(24) $\sum_{i=1}^{9} 2 \cdot 2^{i-1}$ $\quad a_1 = 2; \quad n = 9; \quad r = 2$

$$S_n = \frac{a_1(1-r^n)}{1-r}$$

$$S_n = \frac{2(1-2^9)}{1-2} = \frac{2(1-512)}{1-2} = \frac{2(-511)}{-1} = \frac{-1022}{-1} = \boxed{1022}$$

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(25) $a_1 = -2, \quad r = -2; \quad n = 9$

$$S_n = \frac{a_1(1-r^n)}{1-r}$$

$$S_9 = \frac{-2(1-(-2)^9)}{1-(-2)} = \frac{-2(1-(-512))}{1-(-2)} = \frac{-2(1+512)}{1+2} = \frac{-2(513)}{3}$$

$$S_9 = \frac{-1026}{3} = \boxed{-342}$$

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(26) $a_1 = 2, \quad r = -4, \quad n = 7$

$$S_n = \frac{a_1(1-r^n)}{1-r}$$

$$S_7 = \frac{2(1-(-4)^7)}{1-(-4)} = \frac{2(1-(-16384))}{1-(-4)} = \frac{2(1+16384)}{1+4} = \frac{2(16385)}{5}$$

$$S_7 = \frac{32770}{5} = \boxed{6554}$$