Real-World Applications – Day 1

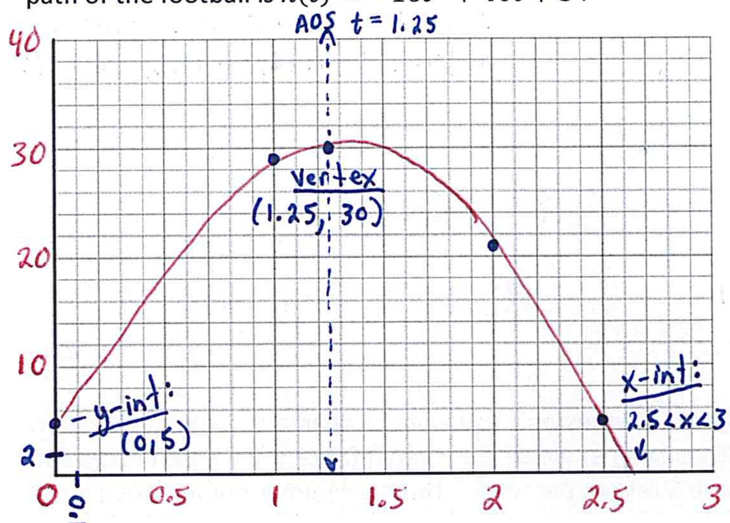
Unit 2B: Quadratic Functions - Modeling

For each of the following:

A. Identify the x-intercept(s) and tell what they mean
B. Identify the y-intercept and tell what it means
C. Identify the maxima/minima of the function and tell what it means
D. Graph and label the function

1. Christian throws a football at 40 feet per second from a height of 5 foot. The equation that shows the path of the football is $h(t) = -16t^2 + 40t + 5$.

Scale: $\frac{\text{y-value of vertex}}{4} = \frac{30}{4} = 7.5$ Round to 10.



Scale: $\frac{\text{Bottom chart value}}{6} = \frac{3}{6} = \frac{1}{2} = 0.5$

AOS:

$t = \frac{-b}{2a} = \frac{-40}{-32} = 1.25$

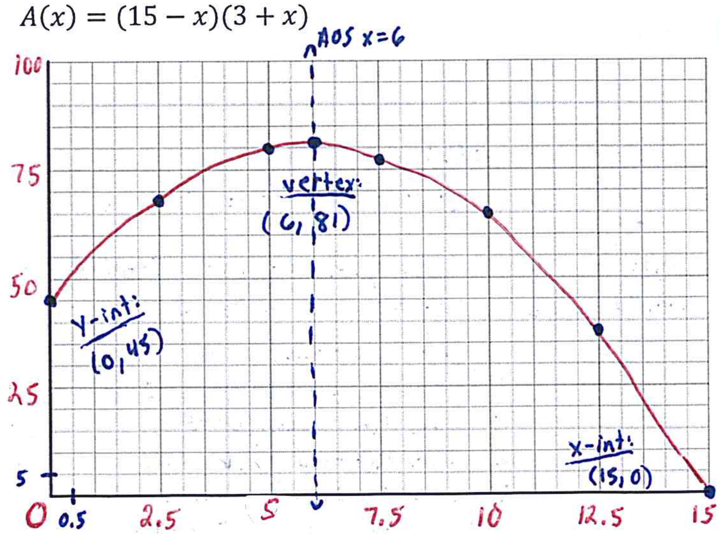
t	$h(t) = -16t^2 + 40t + 5$	$(t, h(t))$
0	$h(0) = -16(0)^2 + 40(0) + 5$	(0, 5)
1	$h(1) = -16(1)^2 + 40(1) + 5$	(1, 29)
1.25	$h(1.25) = -16(1.25)^2 + 40(1.25) + 5$	(1.25, 30)
2	$h(2) = -16(2)^2 + 40(2) + 5$	(2, 21)
3	$h(3) = -16(3)^2 + 40(3) + 5$	(3, -19)
2.5	$h(2.5) = -16(2.5)^2 + 40(2.5) + 5$	(2.5, 5)

Needed to add value to help with graph.

(A) $2.5 < x < 3$ Time it took ball to hit ground.
(B) (0, 5) Release height of ball
(C) 30 Highest ball reaches before falling.

2. A farmer has noticed that some of the wild animals around have been picking through their garden and want to put a fence around it. The area of land that he wants to fence in is given by the equation $A(x) = (15 - x)(3 + x)$

Scale: $\frac{81}{4} = 20.25$ Round to 25



Scale: $\frac{\text{Pos x int}}{6} = \frac{15}{6} = 2.5$

(A) $15 - x = 0$; $+x \quad +x$; $15 = x$; (15, 0)
$3 + x = 0$; $-3 \quad -3$; $x = -3$; (-3, 0)

AOS:

$x = \frac{15 + (-3)}{2} = \frac{12}{2} = 6$

x	$A(x) = (15 - x)(3 + x)$	$(x, A(x))$
0	$A(0) = (15 - 0)(3 + 0)$	(0, 45)
2.5	$A(2.5) = (15 - 2.5)(3 + 2.5)$	(2.5, 68.75)
5	$A(5) = (15 - 5)(3 + 5)$	(5, 80)
6	$A(6) = (15 - 6)(3 + 6)$	(6, 81)
7.5	$A(7.5) = (15 - 7.5)(3 + 7.5)$	(7.5, 78.75)
10	$A(10) = (15 - 10)(3 + 10)$	(10, 65)
12.5	$A(12.5) = (15 - 12.5)(3 + 12.5)$	(12.5, 38.75)
15	$A(15) = (15 - 15)(3 + 15)$	(15, 0)

(B) (0, 45) Started with an area of 45 ft^2

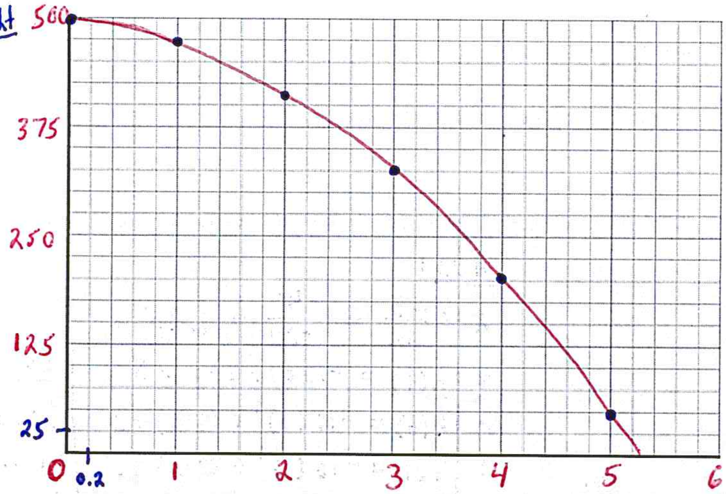
(C) 81 ft^2 is largest possible area after a 6 ft shift.

3. Standing at the top of an office building a man decides to toss a penny over the ledge. The path of the penny after the man throws it from 500 feet with a downward force of 10 feet per second. This is modeled by the equation $h(t) = -16t^2 - 10t + 500$

Scale: $\frac{\text{Initial height}}{4}$

$\frac{500}{4}$

$= 125$



Scale: $\frac{\text{Bottom chart value}}{6} = \frac{6}{6} = 1$

AOS: $t = \frac{-b}{2a} = \frac{10}{-32} = -0.3125$

t	$h(t) = -16t^2 - 10t + 500$	$(t, h(t))$
0	$h(0) = -16(0)^2 - 10(0) + 500$	$(0, 500)$
1	$h(1) = -16(1)^2 - 10(1) + 500$	$(1, 474)$
2	$h(2) = -16(2)^2 - 10(2) + 500$	$(2, 416)$
3	$h(3) = -16(3)^2 - 10(3) + 500$	$(3, 326)$
4	$h(4) = -16(4)^2 - 10(4) + 500$	$(4, 204)$
5	$h(5) = -16(5)^2 - 10(5) + 500$	$(5, 50)$
6	$h(6) = -16(6)^2 - 10(6) + 500$	$(6, -136)$

Ⓐ $5 < t < 6$ The penny hit the ground between 5 and 6 seconds after release.

Ⓑ $(0, 500)$ Initially released from 500 ft.

Ⓒ 500 ft. since the object is thrown down it will not go above 500 ft.

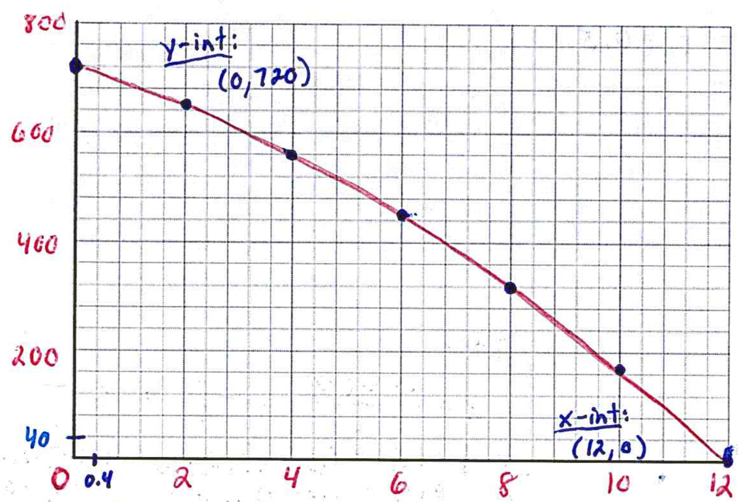
4. You want to run a car-rental business here in Hoopeston. You want to charge \$12 per day to rent a vehicle and believe that you will average 60 rentals per week. For every fifty-cent increase in the rental price, the average business can expect to lose 5 rentals per week. The maximum profit and the amount of increase you should charge is given by the equation $C(r) = (12 + 0.50r)(60 - 5r)$

Scale:

$\frac{720}{4}$

$= 180$

round to 200



Scale: $\frac{\text{Pos x-int}}{6} = \frac{12}{6} = 2$

Ⓐ

$12 + 0.50r = 0$
$-12 \qquad -12$
$\frac{0.50r}{0.5} = \frac{-12}{0.5}$
$r = -24$
$(-24, 0)$

$60 - 5r = 0$
$+5r \qquad +5r$
$\frac{60}{5} = \frac{5r}{5}$
$12 = r$
$(12, 0)$

AOS:

$r = \frac{-24 + 12}{2} = \frac{-12}{2} = -6$

Not able to plot implies we can't make any more \$ without decreasing the current cost.

r	$C(r) = (12 + 0.50r)(60 - 5r)$	$(r, C(r))$
0	$C(0) = (12 + 0.5(0))(60 - 5(0))$	$(0, 720)$
2	$C(2) = (12 + 0.5(2))(60 - 5(2))$	$(2, 650)$
4	$C(4) = (12 + 0.5(4))(60 - 5(4))$	$(4, 560)$
6	$C(6) = (12 + 0.5(6))(60 - 5(6))$	$(6, 450)$
8	$C(8) = (12 + 0.5(8))(60 - 5(8))$	$(8, 320)$
10	$C(10) = (12 + 0.5(10))(60 - 5(10))$	$(10, 170)$
12	$C(12) = (12 + 0.5(12))(60 - 5(12))$	$(12, 0)$

Ⓑ $(0, 720)$ means the current sales per week is \$720 without any cost change.

Ⓒ \$720 is the most we can make. Any increase in cost loses customers and sales.