Solving Quadratic Equations by Completing the Square – Day 1

Unit 2C: Quadratic Functions – Working With Equations

Solve the equation:

1. $x^2 - 8x = 3$

$x^2 - 8x + \_\_\_ = 3 + \_\_\_$

$x^2 - 8x + \left(\frac{-8}{2}\right)^2 = 3 + \left(\frac{-8}{2}\right)^2$

$x^2 - 8x + (-4)^2 = 3 + (-4)^2$

$(x-4)^2 = 3 + 16$

$(x-4)^2 = 19$

$\sqrt{(x-4)^2} = \sqrt{19}$

$x - 4 = \pm\sqrt{19}$

$+4 \quad +4$

$\boxed{x = 4 \pm \sqrt{19}}$

2. $x^2 + 2x = 13$

$x^2 + 2x + \_\_\_ = 13 + \_\_\_$

$x^2 + 2x + \left(\frac{2}{2}\right)^2 = 13 + \left(\frac{2}{2}\right)^2$

$x^2 + 2x + (1)^2 = 13 + (1)^2$

$(x+1)^2 = 13 + 1$

$(x+1)^2 = 14$

$\sqrt{(x+1)^2} = \sqrt{14}$

$x + 1 = \pm\sqrt{14}$

$-1 \quad -1$

$\boxed{x = -1 \pm \sqrt{14}}$

3. $x^2 - 5x - 1 = 4 - 3x$

$+3x \quad +1 \quad +1 \quad +3x$

$x^2 - 2x = 5$

$x^2 - 2x + \_\_\_ = 5 + \_\_\_$

$x^2 - 2x + \left(\frac{-2}{2}\right)^2 = 5 + \left(\frac{-2}{2}\right)^2$

$x^2 - 2x + (-1)^2 = 5 + (-1)^2$

$(x-1)^2 = 5 + 1$

$(x-1)^2 = 6$

$\sqrt{(x-1)^2} = \sqrt{6}$

$x - 1 = \pm\sqrt{6}$

$\boxed{x = 1 \pm \sqrt{6}}$

4. $0 = x^2 - 6x + 3$

$-3 \quad -3$

$-3 = x^2 - 6x$

$-3 + \_\_\_ = x^2 - 6x + \_\_\_$

$-3 + \left(\frac{-6}{2}\right)^2 = x^2 - 6x + \left(\frac{-6}{2}\right)^2$

$-3 + (-3)^2 = x^2 - 6x + (-3)^2$

$-3 + 9 = (x-3)^2$

$6 = (x-3)^2$

$\sqrt{6} = \sqrt{(x-3)^2}$

$\pm\sqrt{6} = x - 3$

$\boxed{3 \pm \sqrt{6} = x}$

5. $0 = x^2 + 7x - 26$

$+26 \quad +26$

$26 = x^2 + 7x$

$26 + \_\_\_ = x^2 + 7x + \_\_\_$

$26 + \left(\frac{7}{2}\right)^2 = x^2 + 7x + \left(\frac{7}{2}\right)^2$

$\frac{26}{1} + \frac{49}{4} = \left(x + \frac{7}{2}\right)^2$

$\frac{104}{4} + \frac{49}{4} = \left(x + \frac{7}{2}\right)^2$

$\frac{153}{4} = \left(x + \frac{7}{2}\right)^2$

$\frac{\sqrt{153}}{\sqrt{4}} = \sqrt{\left(x + \frac{7}{2}\right)^2}$

$\pm\frac{\sqrt{153}}{2} = x + \frac{7}{2}$

$x = \frac{-7}{2} \pm \frac{\sqrt{153}}{2}$

$x = \frac{-7}{2} \pm \frac{\sqrt{9 \cdot 17}}{2}$

$\boxed{x = \frac{-7}{2} \pm \frac{3\sqrt{17}}{2}}$

Factor 153	
1	153
3	51
9	17

6. $0 = x^2 - 3x - 6$

$+6 \quad +6$

$6 = x^2 - 3x$

$6 + \_\_\_ = x^2 - 3x + \_\_\_$

$6 + \left(\frac{-3}{2}\right)^2 = x^2 - 3x + \left(\frac{-3}{2}\right)^2$

$\frac{6}{1} + \frac{9}{4} = \left(x - \frac{3}{2}\right)^2$

$\frac{24}{4} + \frac{9}{4} = \left(x - \frac{3}{2}\right)^2$

$\frac{33}{4} = \left(x - \frac{3}{2}\right)^2$

$\frac{\sqrt{33}}{\sqrt{4}} = \sqrt{\left(x - \frac{3}{2}\right)^2}$

$\pm\frac{\sqrt{33}}{2} = x - \frac{3}{2}$

$\boxed{\frac{3}{2} \pm \frac{\sqrt{33}}{2} = x}$

Factor 33	
1	33
3	11

No perfect square factors.

Solve the equation:

7. $x^2 + 7x + 10 = 0$

$-10 \quad -10$

$x^2 + 7x = -10$

$x^2 + 7x + \_\_\_ = -10 + \_\_\_$

$x^2 + 7x + \left(\frac{7}{2}\right)^2 = -10 + \left(\frac{7}{2}\right)^2$

$\left(x + \frac{7}{2}\right)^2 = -\frac{10}{1} + \frac{49}{4}$

$\left(x + \frac{7}{2}\right)^2 = -\frac{40}{4} + \frac{49}{4}$

$\left(x + \frac{7}{2}\right)^2 = \frac{9}{4}$

$\sqrt{\left(x + \frac{7}{2}\right)^2} = \frac{\sqrt{9}}{\sqrt{4}}$

$x + \frac{7}{2} = \pm \frac{3}{2}$

$x = -\frac{7}{2} \pm \frac{3}{2}$

So...

$x = -\frac{7}{2} + \frac{3}{2}$

$x = \frac{-4}{2}$

$\boxed{x = -2}$

$x = -\frac{7}{2} - \frac{3}{2}$

$x = \frac{-10}{2}$

$\boxed{x = -5}$

8. $x^2 + 10x + 16 = 0$

$-16 \quad -16$

$x^2 + 10x = -16$

$x^2 + 10x + \_\_\_ = -16 + \_\_\_$

$x^2 + 10x + \left(\frac{10}{2}\right)^2 = -16 + \left(\frac{10}{2}\right)^2$

$x^2 + 10x + (5)^2 = -16 + (5)^2$

$(x+5)^2 = -16 + 25$

$(x+5)^2 = 9$

$\sqrt{(x+5)^2} = \sqrt{9}$

$x + 5 = \pm 3$

$x = -5 \pm 3$

So...

$x = -5 + 3$

$\boxed{x = -2}$

$x = -5 - 3$

$\boxed{x = -8}$

9. $x^2 - x = 30$

$x^2 - x + \_\_\_ = 30 + \_\_\_$

$x^2 - x + \left(-\frac{1}{2}\right)^2 = 30 + \left(-\frac{1}{2}\right)^2$

$\left(x - \frac{1}{2}\right)^2 = \frac{30}{1} + \frac{1}{4}$

$\left(x - \frac{1}{2}\right)^2 = \frac{120}{4} + \frac{1}{4}$

$\left(x - \frac{1}{2}\right)^2 = \frac{121}{4}$

$\sqrt{\left(x - \frac{1}{2}\right)^2} = \frac{\sqrt{121}}{\sqrt{4}}$

$x - \frac{1}{2} = \pm \frac{11}{2}$

$x = \frac{1}{2} \pm \frac{11}{2}$

So...

$x = \frac{1}{2} + \frac{11}{2}$

$x = \frac{12}{2}$

$\boxed{x = 6}$

$x = \frac{1}{2} - \frac{11}{2}$

$x = \frac{-10}{2}$

$\boxed{x = -5}$

10. $0 = 3x^2 - 2x - 12$

$+12 \qquad +12$

$\frac{12}{3} = \frac{3x^2}{3} - \frac{2x}{3}$

$4 = x^2 - \frac{2}{3}x$

$4 + \_\_\_ = x^2 - \frac{2}{3}x + \_\_\_$

$4 + \left(-\frac{1}{3}\right)^2 = x^2 - \frac{2}{3}x + \left(-\frac{1}{3}\right)^2$

$\frac{4}{1} + \frac{1}{9} = \left(x - \frac{1}{3}\right)^2$

$\frac{36}{9} + \frac{1}{9} = \left(x - \frac{1}{3}\right)^2$

$\frac{37}{9} = \left(x - \frac{1}{3}\right)^2$

$\frac{\sqrt{37}}{\sqrt{9}} = \sqrt{\left(x - \frac{1}{3}\right)^2}$

$\pm \frac{\sqrt{37}}{3} = x - \frac{1}{3}$

$\boxed{\frac{1}{3} \pm \frac{\sqrt{37}}{3} = x}$

11. $-2x^2 + 14x + 60 = 0$

$-60 \qquad -60$

$\frac{-2x^2}{-2} + \frac{14x}{-2} = \frac{-60}{-2}$

$x^2 - 7x + \_\_\_ = 30 + \_\_\_$

$x^2 - 7x + \left(-\frac{7}{2}\right)^2 = 30 + \left(-\frac{7}{2}\right)^2$

$\left(x - \frac{7}{2}\right)^2 = \frac{30}{1} + \frac{49}{4}$

$\left(x - \frac{7}{2}\right)^2 = \frac{120}{4} + \frac{49}{4}$

$\left(x - \frac{7}{2}\right)^2 = \frac{169}{4}$

$\sqrt{\left(x - \frac{7}{2}\right)^2} = \frac{\sqrt{169}}{\sqrt{4}}$

$x - \frac{7}{2} = \pm \frac{13}{2}$

$x = \frac{7}{2} \pm \frac{13}{2}$

So...

$x = \frac{7}{2} + \frac{13}{2}$

$x = \frac{20}{2}$

$\boxed{x = 10}$

$x = \frac{7}{2} - \frac{13}{2}$

$x = \frac{-6}{2}$

$\boxed{x = -3}$

12. $0 = 3x^2 - 11x + 6$

$-6 \qquad -6$

$\frac{-6}{3} = \frac{3x^2}{3} - \frac{11x}{3}$

$-2 + \_\_\_ = x^2 - \frac{11}{3}x + \_\_\_$

$-2 + \left(-\frac{11}{6}\right)^2 = x^2 - \frac{11}{3}x + \left(-\frac{11}{6}\right)^2$

$-\frac{2}{1} + \frac{121}{36} = \left(x - \frac{11}{6}\right)^2$

$-\frac{72}{36} + \frac{121}{36} = \left(x - \frac{11}{6}\right)^2$

$\frac{49}{36} = \left(x - \frac{11}{6}\right)^2$

$\frac{\sqrt{49}}{\sqrt{36}} = \sqrt{\left(x - \frac{11}{6}\right)^2}$

$\pm \frac{7}{6} = x - 11/6$

$x = \frac{11}{6} \pm \frac{7}{6}$

So...

$x = \frac{11}{6} + \frac{7}{6}$

$x = \frac{18}{6}$

$\boxed{x = 3}$

$x = \frac{11}{6} - \frac{7}{6}$

$x = \frac{4}{6}$

$\boxed{x = \frac{2}{3}}$