

Area of Circles and Sectors

Unit 7: Circles

Find the exact area of the circle with the given radius or diameter.

1. Radius = 10 inches

$$A = \pi r^2$$

$$A = \pi (10)^2 = \boxed{100\pi}$$

2. Diameter = 42 feet

$$A = \pi r^2 = \pi \left(\frac{d}{2}\right)^2$$

$$A = \pi \left(\frac{42}{2}\right)^2 = \pi (21)^2 = \boxed{441\pi}$$

Find the indicated measure.

3. The area of a circle is 58 in^2 . Find the radius.

$$A = \pi r^2$$

$$\frac{58}{\pi} = \frac{\pi r^2}{\pi}$$

$$\sqrt{\frac{58 \cdot \sqrt{\pi}}{\pi \cdot \sqrt{\pi}}} = \sqrt{r^2}$$

$$\boxed{\frac{\sqrt{58\pi}}{\pi} = r}$$

4. The area of a circle is 37 m^2 . Find the radius.

$$A = \pi r^2$$

$$\frac{37}{\pi} = \frac{\pi r^2}{\pi}$$

$$\sqrt{\frac{37 \cdot \sqrt{\pi}}{\pi \cdot \sqrt{\pi}}} = \sqrt{r^2}$$

$$\boxed{\frac{\sqrt{37\pi}}{\pi} = r}$$

5. The area of a circle is 106 cm^2 . Find the diameter.

$$A = \pi r^2$$

$$\frac{106}{\pi} = \frac{\pi r^2}{\pi}$$

$$\sqrt{\frac{106 \cdot \sqrt{\pi}}{\pi \cdot \sqrt{\pi}}} = \sqrt{r^2}$$

$$\frac{2}{1} \left(\frac{\sqrt{106\pi}}{\pi} \right) = 2(r)$$

$$\boxed{\frac{2\sqrt{106\pi}}{\pi} = d}$$

6. The area of a circle is 249 ft^2 . Find the diameter.

$$A = \pi r^2$$

$$\frac{249}{\pi} = \frac{\pi r^2}{\pi}$$

$$\sqrt{\frac{249 \cdot \sqrt{\pi}}{\pi \cdot \sqrt{\pi}}} = \sqrt{r^2}$$

$$\frac{2}{1} \left(\frac{\sqrt{249\pi}}{\pi} \right) = 2(r)$$

$$\boxed{\frac{2\sqrt{249\pi}}{\pi} = d}$$

Find the area of the sectors formed by the given information.

7. Radius = 6 cm

Central Angle is a right angle.

$$A_{\text{sec}} = \frac{m\widehat{AB}}{360} \cdot \pi r^2$$

$$= \frac{90}{360} \cdot \frac{\pi (6)^2}{1}$$

$$= \frac{90 \cdot 36 \cdot \pi}{360} = \frac{3240\pi}{360} = \boxed{9\pi}$$

8. Diameter = 20 in

Central Angle = 60°

$$A_{\text{sec}} = \frac{m\widehat{AB}}{360} \cdot \pi r^2 = \frac{m\widehat{AB}}{360} \cdot \pi \left(\frac{d}{2}\right)^2$$

$$= \frac{60}{360} \cdot \pi \left(\frac{20}{2}\right)^2$$

$$= \frac{60}{360} \cdot \frac{\pi (10)^2}{1}$$

$$= \frac{60 \cdot 100 \cdot \pi}{360} = \frac{6000\pi}{360} = \boxed{\frac{50\pi}{3}}$$

9. Radius = 24 ft

Central Angle = 120°

$$A_{\text{sec}} = \frac{m\widehat{AB}}{360} \cdot \pi r^2$$

$$= \frac{120}{360} \cdot \frac{\pi (24)^2}{1}$$

$$= \frac{120 \cdot 576 \cdot \pi}{360} = \frac{69120\pi}{360} = \boxed{192\pi}$$

10. Diameter = 8 m

Central Angle = 65°

$$A_{\text{sec}} = \frac{m\widehat{AB}}{360} \cdot \pi r^2 = \frac{m\widehat{AB}}{360} \cdot \pi \left(\frac{d}{2}\right)^2$$

$$= \frac{65}{360} \cdot \pi \left(\frac{8}{2}\right)^2$$

$$= \frac{65}{360} \cdot \frac{\pi (4)^2}{1}$$

$$= \frac{65 \cdot 16 \cdot \pi}{360} = \frac{1040\pi}{360} = \boxed{\frac{26\pi}{9}}$$

Find the area of the circle given the central angle and the area of a sector of the triangle.

11. Area of the sector = 17.46 m²
Central Angle = 100°

$$A_{\text{sec}} = \frac{m\widehat{AB}}{360} \cdot A_{\text{circle}}$$

$$17.46 = \frac{100}{360} \cdot A$$

$$\frac{360}{100} \left[\frac{17.46}{100} = \frac{100}{360} \cdot A \right] \frac{360}{100}$$

$$A = \frac{628,560}{10,000} = \frac{7857}{125} = 62.856$$

12. Area of the sector = 39.52 in²
Central Angle = 175°

$$A_{\text{sec}} = \frac{m\widehat{AB}}{360} \cdot A_{\text{circle}}$$

$$39.52 = \frac{175}{360} \cdot A$$

$$\frac{360}{175} \left[\frac{39.52}{100} = \frac{175}{360} \cdot A \right] \frac{360}{175}$$

$$A = \frac{1422720}{17,500} = \frac{71136}{875} = \text{Not a nice dec.}$$

13. Area of the sector = 20.89 ft²
Central Angle = 282°

$$A_{\text{sec}} = \frac{m\widehat{AB}}{360} \cdot A_{\text{circle}}$$

$$20.89 = \frac{282}{360} \cdot A$$

$$\frac{360}{282} \left[\frac{2089}{100} = \frac{282}{360} \cdot A \right] \frac{360}{282}$$

$$A = \frac{752040}{28,200} = \frac{6267}{235} = \text{Not a nice decimal}$$

14. Area of the sector = 23.79 cm²
Central Angle = 80°

$$A_{\text{sec}} = \frac{m\widehat{AB}}{360} \cdot A_{\text{circle}}$$

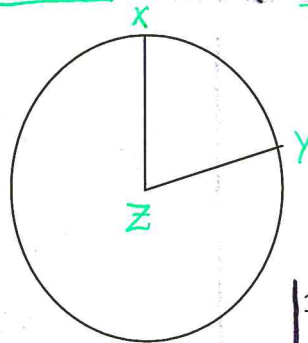
$$23.79 = \frac{80}{360} \cdot A$$

$$\frac{360}{80} \left[\frac{2379}{100} = \frac{80}{360} \cdot A \right] \frac{360}{80}$$

$$A = \frac{856440}{8000} = \frac{21411}{200} = 107.055$$

Mixed Review:

Given circle Z has an area of 124.44 cm². The area of sector XZY is 28 cm². Find the indicated measure.



15. Radius of circle Z.

$$A = \pi r^2$$

$$\frac{124.44}{\pi} = \frac{\pi r^2}{\pi}$$

$$\frac{\sqrt{124.44} \cdot \sqrt{\pi}}{\sqrt{\pi} \cdot \sqrt{\pi}} = \sqrt{r^2}$$

$$\frac{\sqrt{124.44\pi}}{\pi} = r$$

16. Circumference of circle Z.

$$C = 2\pi r$$

$$= \frac{2\pi}{1} \cdot \frac{\sqrt{124.44\pi}}{\pi}$$

$$= 2\sqrt{124.44\pi}$$

17. m \widehat{XY}

$$A_{\text{sec}} = \frac{m\widehat{XY}}{360} \cdot A_{\text{circle}}$$

$$28 = \frac{m\widehat{XY}}{360} \cdot \frac{124.44}{1}$$

$$\frac{360 \cdot 100}{12444} \left[\frac{28}{1} = \frac{m\widehat{XY}}{360} \cdot \frac{12444}{100} \right] \frac{360 \cdot 100}{12444}$$

$$m\widehat{XY} = \frac{1008000}{12444} = \frac{84000}{1037}$$

18. Length of \widehat{XY}

$$\widehat{XY} = \frac{m\widehat{XY}}{360} \cdot 2\pi r = \frac{m\widehat{XY}}{360} \cdot C$$

$$= \frac{84000}{360} \cdot 2\sqrt{124.44\pi}$$

$$= \frac{700}{311} \cdot \frac{2\sqrt{124.44\pi}}{1} = \frac{1400\sqrt{124.44\pi}}{311}$$